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ABSTRACT

This is a condensed report of a study commissioned by the Minister's Advisory Committee on Student Achievement (MACOSA). The study was designed to provide information about current levels of achievement in mathematics among students in Alberta schools and to provide a data base for future assessments. The test was given to third-, sixth-, ninth-, and twelfth-grade students in a 10% random sample of schools. Performance in number facts and computation was satisfactory at all grade levels, but performances in problem solving, geometry, measurement, and consumer mathematics, were unsatisfactory. Boys performed better than girls on all aspects of mathematics and this difference increased as grade level increased. School size also showed a relationship to achievement differences. Fourteen recommendations to MACOSA are reported. (MK)

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ED 172404

Alberta Assessment of School Mathematics

CONDENSED REPORT

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APRIL, 1979

Alberta
Minister's Advisory Committee
on Student Achievement

SE 029 376

Condensed Report

on

Alberta Assessment of School Mathematics

A Study Conducted for

The Minister's Advisory Committee on Student Achievement

by

A.T. Olson, D. Sawada, S.E. Sigurdson

The University of Alberta

under contract to the Planning and Research Branch

of Alberta Education

1979

ABSTRACT

Students at each of grades 3, 6, 9, and 12 responded to 120 questions. About 70 percent of the questions were multiple choice; the remainder were open-ended. Between 54 and 92 percent of the 120 questions were appropriate to the target grade, and the others were review items appropriate to lower grades or preview questions suitable for higher grades. Review and preview items were administered as common items in several grades to provide cross-sectional information.

On May 17, 1978, the tests were administered to a 10 percent random sample of schools consisting of 102 schools offering grade 3; 101 schools offering grade 6; 56 schools offering grade 9; and 50 schools offering grade 12. A form of matrix sampling was employed so that each student did not respond to all 120 test items; instead, different sets of test exercies were randomly assigned to each student.

Performance in number facts and computation (adding, subtracting, multiplying and dividing) was satisfactory at all grade levels, but performances in problem solving, geometry, measurement and consumer mathematics were unsatisfactory. The difficulty with problem solving might be attributed to students' reading abilities as well as to the complex reasoning skills required, but no firm evidence is available to support this speculation. Weaknesses in geometry, measurement and consumer mathematics might be related to the emphasis these areas receive in the Alberta mathematics curriculum. Geometry studies at the elementary level have only recently been increased, the metric system of measurement is relatively new to Alberta school programs, and consumer mathematics, although considered an important skill for daily living, receives little or no attention in the senior high school.

This study found that boys performed better than girls on all aspects of mathematics and this difference became more pronounced as the grade level increased. School size also showed a relationship to achievement differences, and interestingly, the differences interacted with grade levels. Grade 3 students in small schools performed best; in grade 6 the medium-sized school began to emerge as dominant; for grade 9 the medium school showed clear dominance; and at the grade 12 level school size failed to show any significant differences in achievement. The large elementary school quite consistently had the lowest levels of performance. Regarding the type of high school mathematics program, students who had taken Math 30 had the highest scores. It is not surprising that the Math 30 students performed best in the areas where they had received the greatest amount of instruction (number work and algebra) and that there were fewer differences among students in different mathematics programs in areas where the amount of instruction is more equivalent (geometry, measurements, statistics and consumer mathematics).

FOREWORD

The Minister's Advisory Committee on Student Achievement (MACOSA) was established by ministerial order in October 1976 in response to growing concerns expressed by the public-at-large, government, labor, business, students and educators regarding the quality and standards of basic education in Alberta.

MACOSA commissioned a number of studies, primarily to provide basic information for a summary of current levels of achievement in Alberta and to provide baseline data for future assessment. These studies fell into three categories: (1) preliminary studies, (2) achievement studies, and (3) other studies.

This achievement study, Alberta Assessment of School Mathematics, was designed to provide information about current levels of achievement in mathematics among students in Alberta schools and to provide a data base for future assessments.

This report, which represents the findings and conclusions of the researchers, was presented to MACOSA as information.

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PREFACE

In doing the study reported herein the contractors took the position that the assessment of student achievement needs drastic re-conceptualization if the data gathered are to have any substantial and ongoing impact on educational programs and policies. The traditional "tests and measurement" point of view has not provided enough punch to be taken seriously by those who are involved in top level educational decision making. On the other hand, those involved with top level as well as low level decision making in regards to the economy do consult the Consumer Price Index (CPI) as a valid barometer of the Canadian economy. In fact the monthly fluctuations of the CPI are often treated as headline news. For these and other reasons the present study is reported as if a counterpart mathematics proficiency index (MPI) were already in existence. Terminology referring to various mathematical content indices is used throughout the report even though properly speaking the present study does NOT provide index numbers since in order to compute index numbers a baseline year is required for comparison. However, baseline data do not exist since the present study is the first of its kind to be cast as an index in Alberta. Nevertheless the contractors believe strongly that unless studies of the present kind are designed and interpreted as providing indices, their usefulness in educational decision making will be marginal at best. On this basis the contractors have taken liberties in interpreting the present results as providing estimates of index numbers even though only baseline values are provided. We request the indulgence of the readership in this regard.

GLOSSARY

Application
When spelled with a capital "A", this term refers to the third and highest level of thought process involving the organization and reorganization of information so as to present the information to satisfy predetermined specifications.

Comprehension
When spelled with a capital "C", this term refers to the understanding level of thought process involving concepts and generalizations or a transformation of conceptual elements into an equivalent form.

Division
When spelled with a capital "D", this term refers to the following breakdown of public schooling. Division I: grades 1-3; Division II: grades 4-6; Division III: grades 7-9; Division IV: grades 10-12.

Index
A coherently structured set of index numbers. It should be noted that the present study does not by itself provide an index. The assessment must be repeated at some time in the future before an index can be developed. The present study does however provide the base year estimates of performance.

Index (number)
An index number is a statistical device used to make comparisons of one group of related variables to another group. The idea is to compare mathematical performance on items at a given date to the performance on the same items given at a different date. Since the present study is based on data gathered on one date only, it is impossible to compute true index numbers. The study however is designed to provide base year estimates of mathematical performance so that future assessments could calculate legitimate index numbers. In anticipation of future assessments, the term index number is used to refer to current estimates of performance in the hope that such vocabulary will build the expectation for a regular up-dating of estimates resulting in the provision of true index numbers for mathematical proficiency.

Knowledge
When spelled with a capital "K", this term refers to the simplest level of thought level involving simple recall, definition of terms, computation.

Mathematics Program Variable

The Mathematics Program Variable is used to define the mathematical backgrounds of grade 12 students. It is defined in terms of the last mathematics class that students have completed or are presently completing. Students defined their mathematics background using one of the following six alternatives: One Mathematics course, Mathematics 20, Mathematics 23, Mathematics 25, Mathematics 30, or Mathematics 33. This variable was used in the data analysis.

Matrix sampling

A sampling technique in which both students and test items are sampled. In conventional sampling, only students are sampled. In the current study, no student writes all of the items in a given test battery. Rather, he writes only 20 or 30 items. Thus both students and items are sampled at each occasion of testing.

School size

At each grade level, the sample schools were categorized as small (s), medium (m) or large (l) in order to analyze the performance data in relation to school size. The categorization was done such that at each grade level roughly one third of the schools fell in each category.

Target item

Each test item included in the assessment was based on content relevant to a particular Division. An item was said to be targeted at a particular Division if it was based on content specified for that Division by current curriculum guides for Alberta, the consumer items in Division IV being an exception. A target item for a particular Division is thus an item targeted at that Division.

PART I

BACKGROUND

1 Introduction and Overview

1.1 Origin and Purpose

On January 25, 1977 the Minister's Advisory Committee on Student Achievement (MACOSA) through the Planning and Research Branch of Alberta Education issued a call for proposals that would deal with the assessment of current levels of mathematics skills in Alberta schools over four grade levels, 3, 6, 9, and 12. According to that document

> The purpose of the study will be to provide information concerning the current levels of mathematics skills and knowledge as demonstrated by students in Alberta schools and to provide a base for future assessments.

Just what type of information and in what form it was to be provided and the use to which it would be put was not clarified in the document. Apparently part of the task of the proposal writer was to specify just what type of information was required and what form that information should take in order to fulfill the purpose as stated.

1.2 The Problem

In viewing and reviewing the state of assessment in North America and in Europe, it became clear to the contractors that

1. the type of information that was desired had to be understandable and face valid,

2. the form of the information had to be similar to that provided for similar purposes in other aspects of human endeavor so that the meaning and interpretation of the information could be arrived at in relatively well known and familiar ways by anyone interested in mathematical competence.

Thus in the same way that the Consumer Price Index allows the layman as well as the economist to monitor the current cost of living, so too should the information provided by this study and subsequent studies, allow laymen and mathematics educators to monitor the current levels of mathematical competence of Alberta students.

In a nutshell, then, the problem of the study was defined as follows:

1. to provide a comprehensive and coherently structured set of index numbers that would faithfully reflect current levels of mathematical competence of Alberta students, and

2. to design a simple yet effective scheme whereby these index numbers could be updated on a regular basis in order that a valid monitoring of student mathematical competence could be provided through a set of indices.

In defining the problem in the above way, the traditional test item was cast in a new role and in new light: at one extreme an individual test item could serve to estimate a single index number; more likely, however, several test items would be combined in the form of arithmetic means to estimate index numbers for various aspects of mathematical competence. Under this interpretation, an individual test item such as "2 + 6 = ?" would have something in common with a loaf of bread: the test item contributes to the estimation of what might be called the Mathematical Proficiency Index just as the price of a loaf of bread contributes to the estimation of the Consumer Price Index. The notion of an index undergirds much of the current methodology of assessment and in particular forms the basis of the National Assessment of Educational Progress (NAEP) in the United States. It, as well, forms the conceptual and statistical basis for the present study. As such, much of the value of the current assessment will remain untapped if regular updatings of the index numbers are not provided in the near future, since only by obtaining a series of index numbers (over time) can one obtain indices by computing ratios with the index numbers provided by the current assessment used as the base.

1.3 Approach to the Problem

The major point of departure of this assessment is the use of the notion of an index as the proper statistical quantity to relay information regarding the status of student achievement. Without such an orientation, the results of this and other assessments like it will disappear into the archives leaving little or no impact on the educational world. With it, the continuity and base line comparisons provided should ensure the relevance and usefulness of the present results for now and in the future. The design of the present assessment was completed with the notion of an index given high priority.

The approach to the study is summarized in the following pages under five headings. More detailed specifications can be found in the Technical Report.

1.3.1 Survey Variables

The major variable was the set of index numbers to be developed in a structured and coherent manner. An individual index number was estimated by the percentage of respondents of a given

type who gave the correct answer on a particular test item (exercise). The structure and coherence of these estimates was provided by the use of two types of partitioning variables: (1) mathematical content specifications done according to level of thought process and "targeted" grade level within the Alberta curriculum, and (2) personological/demographic variables.

The mathematical content specification variables took the form of a 3-dimensional matrix with mathematics content (number, algebra, geometry, measurement, statistics, and consumer mathematics) forming the first dimension; level of thought process (Knowledge, Comprehension, and Application) forming the second dimension; and targeted grade level (Division I for grades 1-3, Division II for grades 4-6, Division III for grades 7-9 and Division IV for grades 10-12) forming the third dimension. The resulting matrix, called the Mathematics Content Specification Model, is shown in Figure 1-1. For Divisions I, II and III the content domain was taken to be defined by the various curriculum guides published by Alberta Education for use in Alberta schools. For Division IV the content domain was taken to be reflective of the mathematics that would be useful in daily life as well as the various mathematics programs given in secondary schools.

The personological/demographic variables consisted of sex, grade level, school size, and at the high school level the type of mathematics program taken. These variables were included primarily to provide means of partitioning the percentage estimates into identifiable sources such as grade, sex and so on so that comparisons and contrasts could be made to aid in the interpretation of the indices. As well, the variable of grade level was used in a special way to provide a cross-sectional view of growth. More particularly, after a test item was selected according to the mathematical content specification model, a decision had to be made as to what grade level(s) the item would be given. If given at more than one grade level, the item was called a common item and was therefore useful for tracing growth. Thus a given exercise (test item) could be classified according to targeted Division and as well according to its range. A special ordered pair notation was used to convey this information: an item designated (II, I-III) was targeted at Division II but was included in the item battery for Divisions I, II and III. Such an item would be useful for tracing growth over grades 3 through 9. As well, it was assessing content encountered in the curriculum in grades 4-6.

From a conceptual/pedagogical point of view, the notion of target and range for common items permitted the identification of three types of items as follows:

1. Preview items. If an item was given at a Division lower than its target, it provided "preview" information on that content.

2. Target items. All items targeted at a given Division were given at that Division. Target items essentially provided information on how students performed on mathematical concepts and skills specified for their level by the Alberta Program of Studies.

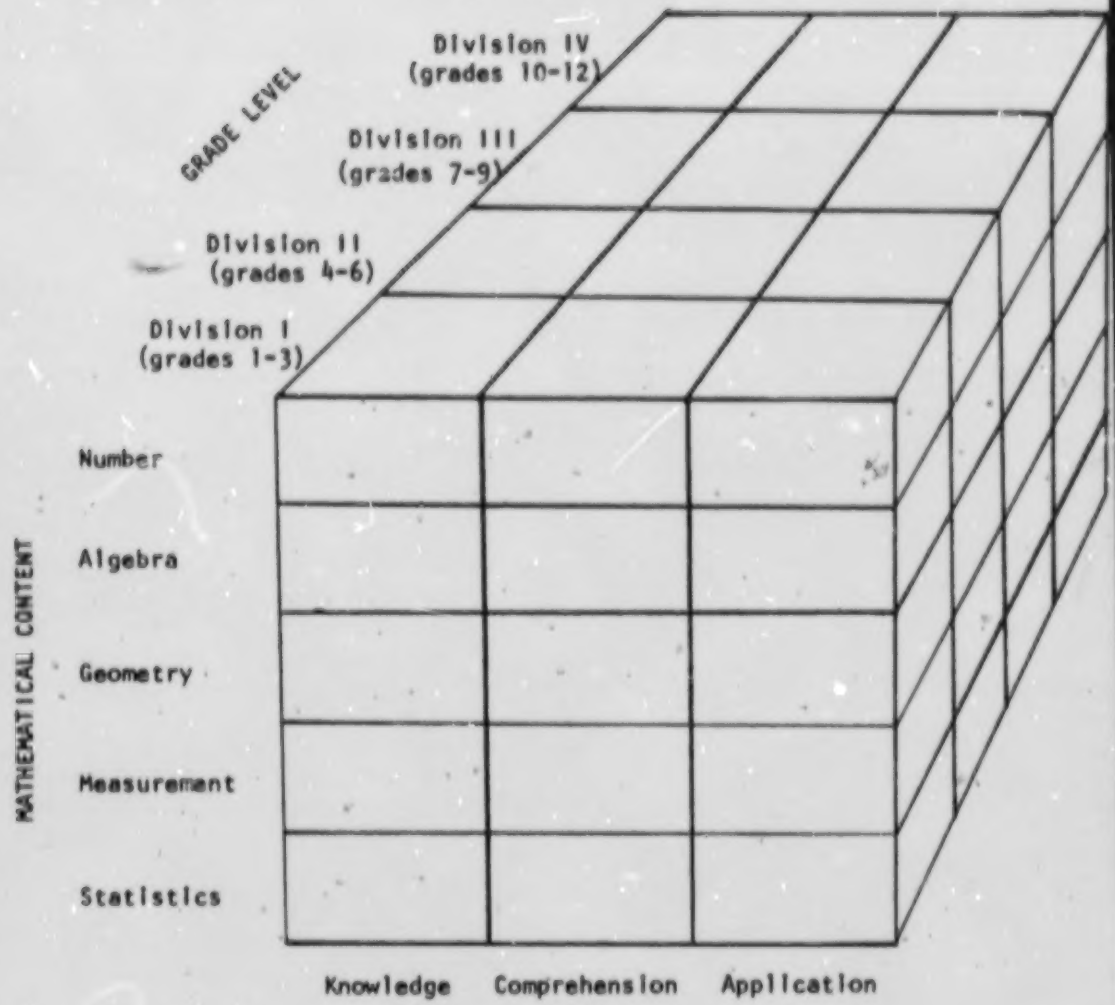


Figure 1-1 Mathematical Content Specification Model.

3. Review items. If an item was given at a Division higher than its target, it provided "review" information on how students performed on material they should have mastered at least three years earlier.

The variables mentioned thus far all relate to ways of structuring the estimates of the index numbers. As well, and for its own sake, student background information of the following kinds was collected: (1) class time devoted to mathematics, (2) use of hand-held calculators, (3) attitude towards mathematics, and (4) usefulness of mathematics as perceived by students.

1.3.2 Methods of Data Collection

The complete battery of items given at a particular grade level (3, 6, 9, or 12) consisted of 120 exercises. The make-up of each battery is indicated in Figure 1-2 in which the number of items classified by content and targeted Division is shown. For example, the item battery for Division I contained 64 exercises on number; 48 of these were targeted at Division I, 11 at Division II, 5 at Division III, and 0 at Division IV. In fact, as shown in the figure, no items in the Division I battery were targeted at Division IV. On the other hand, six items in the Division IV battery were targeted at Division I, in keeping with the guideline that for purposes of assessment, review information is more valuable than preview information.

In selecting the 120 items for each Division, two rounds of screening by classroom teachers took place. In round 1 eight teachers, two each at grades 3, 6, 9, and 12, examined approximately 1500 test items selected from similar assessments done in Canada, U.S.A. and Great Britain. The criteria of clarity, fairness and representativeness were used by the teachers to rate each item. In round 2, eight other teachers from a rural area in Alberta examined the approximately 1200 items rated acceptable or better by the round 1 teachers. This second set of teachers assessed each item according to the range for which it would be useful. On the basis of the information provided by these two rounds of item validation, the contractors were able to select the 120 items for each Division battery. As well, the Steering Committee examined each item in detail and made comments for revision. Since many items were common to more than one battery only 275 different items were needed, these 275 being distributed over the four Divisions.

To assist in the data collection, the 120 items for each Division were packaged into booklets. At Divisions I, II, and III, the booklets contained twenty items each. At Division IV the booklets contained thirty items each. Figure 1-3 gives information on the format of the booklets at each Division. Over all, approximately 70% of the exercises were multiple choice in format, the remaining 30% being open-ended.

Division	Content	Target Division I	II	III	IV	Total
I	Number	49	10	5	0	64
	Algebra	9	2	1	0	12
	Geometry	20	2	4	0	26
	Measurement	10	2	0	0	12
	Statistics	4	1	1	0	6
	Total	92	17	11	0	120
II	Number	14	25	12	2	53
	Algebra	4	7	5	1	17
	Geometry	8	14	8	2	32
	Measurement	3	6	2	1	12
	Statistics	1	3	2	0	6
	Total	30	55	29	6	120
III	Number	3	11	15	6	35
	Algebra	4	7	21	9	41
	Geometry	2	7	11	6	26
	Measurement	2	3	5	2	12
	Statistics	1	1	3	1	6
	Total	12	29	55	24	120
IV	Number	1	1	8	8	18
	Algebra	3	6	12	27	48
	Geometry	1	2	5	10	18
	Measurement	1	1	3	7	12
	Statistics	0	1	2	3	6
	Consumer Math	—	—	—	18	18
	Total	6	11	30	73	120

Figure 1-2 Content Sampling Matrices per Division
(The entries refer to number of items)

Division	Booklet letter	No. of items	No. of open ended items	No. of multiple choice items	Separate answer sheet
I	A	20	20	—	No
	B	20	20	—	No
	C	20	—	20	No
	D	20	—	20	No
	E	20	—	20	No
	F	20	—	20	No
II	A	20	20	—	No
	B	20	20	—	No
	C	20	—	20	Yes
	D	20	—	20	Yes
	E	20	—	20	Yes
	F	20	—	20	Yes
III	A	20	13	7	No
	B	20	12	8	No
	C	20	—	20	Yes
	D	20	—	20	Yes
	E	20	—	20	Yes
	F	20	—	20	Yes
IV	A	30	27	3	No
	B	30	—	30	Yes
	C	30	—	30	Yes
	D	30	—	30	Yes

Figure I-3 Booklet Format per Division

Use of the booklet format was intended to minimize the amount of time any given student had to spend writing since only one booklet was completed by any one student, a task that required no more than forty minutes. Two rounds of pilot testing took place in the Edmonton area and revisions in the booklets were made on the basis of the information obtained.

The sample of students was selected by the Student Evaluation and Data Processing Branch of Alberta Education using the methods of probability sampling. In essence, every student attending schools supported by public funds had an equal chance of being selected. The sample consisted of 102 schools offering grade 3; 101 schools offering grade 6; 56 schools offering grade 9; and 50 schools offering grade 12. The technical details of the matrix sampling scheme are fully described in the technical report.

The test materials were packaged in classroom lots and mailed to the sample schools. Each classroom package contained approximately equal numbers of each booklet for the given grade level. The number of students writing each booklet at each grade level is shown in Figure 1-4. The return rate from the schools was 100% - i.e. return packages were received from all of the schools in the sample. However, on the day of the testing (Wednesday, May 17, 1978) a small portion of students did not attend school. In comparing returns to the number of students in each class as supplied by principals, the rate of return from students was 95% at grade 3, 90% at grade 6, 93% at grade 9, and 73% at grade 12.

1.3.3 Methods of Analysis

Estimates for the index numbers were simply the proportion (percentage) of a given type of respondent giving the correct response to a given exercise. Types of respondents were defined by the personological/demographic variables listed under 1.3.1. Thus for an individual exercise, the estimate is the percentage of correct response over a given type of respondent. For sets of items grouped according to mathematical content specifications, the estimate is simply the arithmetic mean (average) of the single item estimates. No weighting scheme was used or needed to be used in calculating the arithmetic means since the number of items assigned to a given mathematical topic was in proportion to its emphasis in the curriculum at that Division, as specified by the various guides issued by Alberta Education for use by schools.

1.3.4 Utilization of Results

This study was done under contract for the Minister's Advisory Committee on Student Achievement (MACOSA). What MACOSA or the Minister himself will do with the results is not known by the contractors. What the contractors do know is that the results will be made public. It is for this reason that the notion of an index was dominant in the conception of this assessment. The expectation is that the present study is the beginning of something that might be called the

SAMPLE RESOLUTION

DIV I:	ALL	BOYS	GIRLS	S SCHOOL	M SCHOOL	L SCHOOL
BOOK A	563	282	281	149	245	169
BOOK B	573	299	270	157	248	168
BOOK C	570	294	265	159	248	163
BOOK D	566	281	282	150	255	161
BOOK E	548	300	247	144	243	161
BOOK F	553	270	278	147	244	162

DIV II:	ALL	BOYS	GIRLS	S SCHOOL	M SCHOOL	L SCHOOL
BOOK A	520	284	236	150	181	189
BOOK B	518	258	259	148	182	188
BOOK C	501	252	248	144	182	175
BOOK D	511	258	251	154	181	176
BOOK E	511	267	244	157	178	176
BOOK F	507	258	249	147	179	181

DIV III:	ALL	BOYS	GIRLS	S SCHOOL	M SCHOOL	L SCHOOL
BOOK A	542	270	271	96	143	303
BOOK B	543	271	272	101	142	300
BOOK C	522	271	247	96	132	294
BOOK D	531	261	268	96	139	296
BOOK E	525	251	269	94	140	291
BOOK F	531	276	251	93	139	299

DIV IV:	ALL	BOYS	GIRLS	S SCHOOL	M SCHOOL	L SCHOOL
BOOK A	877	424	422	210	228	439
BOOK B	855	406	449	201	227	427
BOOK C	857	405	450	206	226	425
BOOK D	855	411	439	207	219	429

Figure 1-4

Mathematics Proficiency Index (MPI) to be used and interpreted by a wide cross-section of the public in much the same way as the Consumer Price Index (CPI) is used and interpreted by anyone interested in the economy. The provision of index numbers by the present assessment should mark the beginning of a new level of awareness if government decides to make the MPI a reality.

1.3.5 Desired Precision

In studies of the present type it is desirable to know what sort of confidence can be placed on the estimates of the index numbers, i.e. on the percentage of success on an individual exercise. In particular, it is desirable to know whether there is a significant difference between any two percentages. The sample for the study was chosen so that any difference of 9 or more percentage points would be significant at the 0.01 level. For example, if grade 3 students had a 48% success rate on a given exercise whereas the grade 6 students performed at the 58% level, such a difference could be termed a real difference with 99% confidence. In like manner, the sampling scheme ensures that differences exceeding 6 percentage points can be said to be significant at the 0.05 level. The details of the sampling scheme that delivers the results with this precision are given in the technical report.

1.4 Time Line for the Study

The study was carried out in seven phases. These phases are briefly described below since they provide a useful way of summarizing the study as a whole.

1.4.1 Design (April - May, 1977)

Detailed plans regarding

1. sampling
2. mathematical content specifications
3. item validation procedures
4. test piloting procedures
5. data collection scheme
6. data analysis plan
7. final report

were drafted.

1.4.2 Item Specification and Collection (May - September, 1977)

A large bank of test items was collected and classified according to the content-levels matrix developed in the first phase. Released items from numerous state, provincial, national and international mathematics assessments served as the source.

1.4.3 Item Validation (September - December, 1977)

First a group of eight mathematics teachers, two from each of grades 3, 6, 9, and 12, examined all of the items that had been collected and classified according to the content-levels matrix in phase 2. The teachers examined each item for face validity, clarity, and relevance to the Alberta scene.

A second group of eight teachers took the items which had been screened by the first group and indicated at what grade levels it would be useful and feasible to use the item. In essence this group of teachers helped to identify which item would be useful for providing cross-sectional growth information.

1.4.4 Booklet Development and Pilot Testing (January - April, 1978)

Specifications were drawn up regarding the percentage distribution of items over the content areas (number, algebra, geometry, measurement, statistics, and consumer mathematics (Division IV only)) for each Division separately.

As well, specifications were drawn up for the common items indicating how many items would be given at more than one Division, which particular Divisions the item would span, at which Division the item would be targeted, and finally what content would be covered by the item. The details of the planning and the final solutions achieved are described in detail in the technical report.

The total number of items for all four Divisions was 275. Approximately half, 131 to be precise, were common items (given at more than one Division). The remaining 144 were unique items (given at only one Division). Each Division received 120 items, some unique, and some common. These 120 items were divided into six booklets of twenty items each at Divisions I, II, and III; at Division IV, four booklets of thirty were developed.

The booklets underwent two stages of pilot development before final arrangements were made regarding printing and administration procedures.

1.4.5 Data Collection (May, 1978)

All data collection was done via the mail with backup communication provided by long distance telephone. Test packages were mailed by May 5, 1978 to all of the schools in the sample. Postage paid return shipping envelopes were included with every package.

1.4.6 Data Analysis (June, 1978)

Approximately 25% of the items were open ended, the remaining 75% being multiple choice. At the Division II, III and IV levels, the multiple choice items required the use of specially prepared optical scoring answer sheets. The multiple choice items at Division I did not require the use of a separate answer sheet. Two experienced mathematics teachers were hired to score the non-optically-scored booklets. Special programs were written by Computer Services of The University of Alberta to carry out the analyses, the results of which are presented in Part II.

1.4.7 Report Writing (June - July, 1978)

A preliminary report was presented to the Steering Committee on June 21, 1978. Discussions at that time settled the details of the final report form.

1.5 Findings

1.5.1 Content

Number. The results on number showed a great deal of variability with performances ranging from a low of 11% on some items to a high of 99%.

At the Division I level strong performances were obtained on the fundamental skills of primary school mathematics -- performances ranging between 91% and 98% were given on items dealing with cardinal number of a set, reading 3-digit numbers, counting by tens, ordering 4-digit numbers, and the basic facts of addition and subtraction. Rather weak performances were obtained on exercises dealing with rational numbers -- performances ranged between 11% and 72%. At the low end were simple decimal tasks asking that 0.3 be read as three tenths, and at the high end were tasks dealing with recognizing common fractions.

At Division II the strong start made on the fundamental skills was continued with performances in the 90's given on exercises dealing with basic facts of multiplication and division, adding

and subtracting and multiplying multi-digit numbers, reading 5-digit numbers, and rounding to the nearest ten. As with Division I, the work with rational numbers was weak both in common fraction form or decimal form. As well work with integers gave low performances.

For Division III the strong areas for Divisions I and II remained strong. This was particularly true for whole number work. In addition the exercises on place value resulted in strong performances (typically 85%-95%). As well, performances in the 80's were given on whole number applications. Somewhat mediocre performances were given on percent and integers and as at the Division I and II levels, the results on common and decimal fractions failed to reach the level of whole number work although computing with decimal fractions rose significantly.

For Division IV the picture remained pretty well the same as for Division III. Division IV continued to do well on the exercises done well by Division III but failed to improve significantly on the weak areas of Division III.

Algebra. The algebra program receives little emphasis before Division III. The first two Divisions, however, showed some understanding of some fundamental algebraic concepts. Over 80% of the students in Division I showed an understanding of the use of the box "□" in a simple algebraic sentence while two-thirds of them could translate an English statement into a sentence containing a box. Perhaps it is encouraging that one-third indicate an understanding of the concept "more than" and the accompanying symbolism ">". By Division II, two-thirds of the students are using this idea on inequality correctly, while the last two Divisions show 90% correct usage.

The focus of the Division II algebra was verbal problems. Simple problems containing 3 or 4 sentences and one or two operations were answered correctly by less than 75% of the students. Division III shows only slightly better on these problems. Over half of Division II students can solve a simple linear equation involving "x" but all three of the first Divisions show up poorly in solving an equation when x is negative.

Division III does not show well on simple linear expressions and where the answer requires the use of a variable, they show up very poorly. On standard "age" problems Division III does well but any variation making the problem slightly unusual brings about marked drops in the scores. Only half of Division III students understand the linear function.

Division IV does well on simple algebra involving equality, inequalities, simple verbal problems, evaluating polynomials, and simultaneous linear equations. About 75% can interpret a formula involving a linear function. There seems to be several areas in mathematics that can be dealt with by half of the students: quadratics, graphs of linear equations, exponents, division of polynomials, and solving rational expressions. There are also several areas where students perform at less than 50% and indeed much lower: conics, composite functions, exponential

functions and trigonometry. The Division IV results are characterized by poorer responses to problems where answers are in terms of variables and depressed scores where problems involve something slightly different.

Geometry. Geometry, together with number and algebra, is a fundamental part of any mathematics program. The amount of emphasis that it receives probably varies greatly from school to school but it probably is included in all school mathematics programs from the early grades and on.

It seems safe to conclude that students do well enough on topics that receive emphasis in the schools. On the other hand they do less well on topics that are not emphasized. A case in point occurs with some exercises related to points, lines, planes and space. One of these exercises in two dimensions had percentages of correct responses of 80, 83 and 88 at Divisions II, III, and IV, respectively, while a similar exercise in three dimensions had percentages of 34 and 36 at Divisions III and IV. Other weak areas, probably because of a lack of exposure, were transformations and symmetry.

Performance levels were relatively high on the following topics: angles, triangles, circles and congruence. In contrast, the following topics were not handled nearly as well: similarity, line relationships (parallelism and perpendicularity), and polygons.

In a number of exercises students seemed to be misled by irrelevant perceptual cues, e.g., a square supported on one of its corners was not recognized by a significant percentage of students. The application exercises, many of which required high verbal decoding skills, were difficult for many students as well.

Measurement. Most of the measurement exercises were simple Knowledge items which tested familiarity with the metric system or the application of metric units in geometric settings. The telling of time, figuring with money, and measuring length had strong beginnings in Division I and rose to 90% performances by Division II. Questions on perimeter, area and volume fared less well resulting in mediocre performances even at Division IV (70%). In particular, exercises dealing with interrelationships among length, area and volume were very poorly handled (30-40%) at Divisions III and IV. As well, items dealing with interrelationships among metric prefixes showed that over one quarter of grade 12 students aren't familiar with them.

Statistics. All of the items in statistics were at the Comprehension or Application level. Division I scored high on picture graphs, less well on bar graphs and about 40% on line graphs with only one-third being able to construct a bar graph. Division II does well on the bar and line graph and 50% on the circle graph. A problem based on a multiple-line line graph showed half of the students as being able to do it. Division III follows the trend with almost 80% responding correctly to the circle graph. Problems based on graphs bring in slightly lower

performances. Interpreting a line graph showed high levels of performance from both Division III and IV. The latter Division did poorly on a simple probability question and very poorly on a problem using elementary statistical concepts. In all cases orderly and consistent growth across Divisions was noted.

Consumer Mathematics. The inclusion of eighteen exercises skills in consumer-related problems was a unique feature of the Division IV. battery. The exercises were all classified as Applications of mathematics.

The performances were not high with a range of percentages of correct responses from 2 to 85 with a mean of 51%.

Consumer related mathematics is not emphasized in the high school mathematics program. Consequently performance levels were not high, particularly on items that required a modicum of knowledge about specialized terminology e.g. margin of profit, mill rate, etc. This part of the assessment was recognized as not being related to the curriculum but it was argued that these consumer-related skills and knowledge are required of adults and so it was desirable to know how well grade 12 students fare.

Performance levels were highest on the following topics: comparative shopping, interest calculations, consumer credit, payroll, taxation, and discount computations. These are probably the most prevalent consumer related problems that people must face.

In contrast, performance levels on the following topics were not as high: profit and loss, banking, credit card statements, insurance, investments, and real estate.

1.5.2 Other Variables

Sex. Over the years it has become the accepted belief that boys achieve better than girls in mathematics. The results of the present study show conclusively and unambiguously that the old belief is valid for Alberta in 1978. Not only did the boys do better on knowledge items but the difference between sexes became stronger for the Comprehension and Application items. Furthermore the sex difference became more pronounced with each increasing grade level resulting in supplements of 5 percent or more in favor of boys at grade 12.

School Size. At each Division, school were classified as small, medium and large. As with sex, school size showed significantly in accounting for achievement differences. Interestingly, the differences interacted with grade level. For Division I, the small school (enrolment under 30), showed the best and quite consistently so over the different content areas. For Divisions II the

medium school (enrolment between 31 and 70) began to emerge as dominant and in Division III the medium school (enrolment between 41 and 101) showed clear dominance. Somewhat surprisingly, school size failed to make any difference at the Division IV level. Perhaps surprising was the fact that the large elementary school (enrolment over 70) was quite consistently the poorest performer. Since most of the large elementary schools occur in centres of large population it is essentially the city elementary schools that performed less well than the small (and usually rural) counterpart.

Grade 12 Mathematics Program Variable. At grade 12 the purpose of the assessment was to measure general proficiency in mathematics. This implied that the assessment not be tied to a particular mathematics curriculum at grade 12.

Although the assessment was to measure general proficiency it was obvious that performances would still vary as a result of differences in mathematical programs. Consequently student mathematical programs was used as a variable in the analysis of the data. The mathematical programs were defined in terms of the following six alternatives: one mathematics course, Math 20, Math 23, Math 25, Math 30, and Math 33.

As expected the highest performance levels came from the Math 30 sample. Next highest came from Math 20 and Math 33. Finally, the three remaining alternatives did least well, i.e., Math 23, Math 25, and one mathematics course.

The greatest differences in performance across mathematical programs occurred on exercises related to number and algebra. This result seems reasonable because it is in these areas where the Math 30 student receives the greatest preparation.

Lesser differences in performance occurred on exercises related to geometry, to measurement, to statistics and to consumer mathematics. These results probably reflect less difference in mathematical preparation on these topics.

Student Background Information. Our survey of student backgrounds showed that student use of calculators outside the classroom is far higher than inside the classroom. Only at Division IV is the amount of usage getting close.

At grade 3 one in three students find mathematics their favorite subject while this is reduced to one in seventeen at grade 12. And while most students think mathematics is valuable, only 33% of grade 12 students find it "very valuable". Such a trend seems almost to be inevitable in that the grade 12 student is becoming exposed to more attractions.

The class time spent on mathematics is quite variable at Division I and II ranging from 35 to 60 minutes per day. Division III has a smaller range with most schools at either 40 or 50 minutes. On the average the amount of time spent on mathematics decreases significantly from Division II to Division III.

1.6 Recommendations to MACOSA

1.6.1 Main Recommendations

The Contractors strongly recommend that:

1. Alberta Education take immediate steps to make the assessment an on-going endeavour and in this way establish a Mathematics Proficiency Index (MPI) for Alberta.

2. Alberta Education develop a set of terms of reference for the MPI through the conduct of a needs assessment study dealing with mathematics proficiency in Alberta.

3. Access to all the reports written by the contractors be made available to all bodies making curricular decisions in Alberta. In particular the Condensed Report should be given as wide circulation as possible. Interpretative panels should be set up with a view to making the information in the reports more easily understood by a wider variety of interested people.

4. Computerized item banking be developed and maintained.

5. All the data gathered by the contractors be made available to any bonafide researcher interested in the information providing anonymity of students and schools be protected.

6. The focus of the Division IV test battery be more on "school leaving" competencies rather than the various senior high school mathematics programs.

7. The variable of sex be subjected to further study with a view to understanding the reasons for the lower performance of girls in Alberta. Such study should result in short range and long range plans to assure that girls have an equal opportunity to acquire competence in mathematics, a competence, the absence of which, acts in many explicit as well as subtle ways in preventing women from having the same freedom as men in determining their occupational as well as intellectual destiny.

8. The variable of school size be examined in a much more explicit and controlled fashion than was possible in a survey. The results of the present study suggest that the large school, particularly at the lower grade levels, may not be as effective an organization for delivering mathematics instruction as smaller units. This general issue should be subjected to

an intensive observational assessment to determine the reasons for the differential effect of school size.

1.6.2 Procedural Recommendations

9. Procedures for developing additional items should be undertaken as early as possible. The need for new items is especially felt at Division I.

10. The common items that are used at more than one Division should be continued.

11. The number of items used at each Division should be increased from 120 to 150. The use of open-ended items for certain types of questions should also be continued.

12. The teacher validation-of-items procedures that were used were very effective and should be made a part of any future assessment.

13. The test administration procedures used were adequate and should be used for the next assessment.

14. The background variables -- attitude, value of mathematics, usefulness of mathematics -- and the "other variables" -- sex, size of school, and mathematics program at grade 12 -- should be included in the next assessment.

1.6.3 Background to the Recommendations

Recommendation 1 develops out of the original statement of the problem. Greater understanding of the effects of the mathematics curriculum will be achieved as we examine how these effects change over time. The Mathematics Proficiency Index would use the current assessment results as baseline data. The actual index could consist of different indices for the various curriculum strands or could be reduced through appropriate weightings to a single index. Recommendation 4 follows as a consequence of this recommendation since it is essential to use a large portion of identical items from one assessment to the next. Computerized item banking is extremely convenient for information across many Divisions on a particular item.

Recommendation 2 arises from a concern about the usage of the information. If knowledge were available as to who was to use the information, the whole structure of the item battery could be focussed appropriately. In particular, Recommendation 6 reflects a concern about the mathematical competence of the early adult. Knowledge of "school leaving" competencies can have implications for the structure of the mathematics curriculum at all levels.

Recommendation 3 is made from a belief that the information collected is very valuable to classroom teachers and every effort should be made to encourage them to make use of it. Interpretative panels are at liberty to make judgements based on their own points of view. These judgements are valuable but should not be confused with the information provided in the assessment reports.

Recommendation 5 suggests that Alberta Education be the repository of all the data collected. Many interesting studies of the data are possible beyond the current study. Alberta Education should encourage researchers to engage in such studies.

Recommendation 7 is concerned with the possibility that girls may not be treated fairly in Alberta in terms of equal opportunity to learn mathematics. It has become clear in recent years that girls in the U. S. are at a disadvantage in mathematics due in large part to reasons beyond their control. For example, it has been found that teacher referrals (Gregory, 1976) as well as standardized achievement tests (Tittle, 1974) are sex biased. Such bias should be studied in Alberta to determine its nature and extent. Indeed, since the mandate of the present study called for the use of existing test items insofar as they were appropriate to the provincial course of studies, over 90% of the items were selected from assessment studies conducted in recent years in the U. S. Thus the bias that Tittle (1974) found in standardized achievement tests may also pervade test items used in American assessments. The possible bias present in the present study is thus evident. Further study of sex bias in mathematics assessment is thus highly recommended, not only to control for it in assessments of this kind, but to fully document the possible unfair treatment that girls may be subjected to by the school systems of Alberta.

Recommendation 8 is concerned with the differences concommitant with large school size which seem to detract from mathematics learning. Discovering what these concommitant differences are could result in programs and organizational changes that might produce improved learning in all schools.

Recommendations 9 through 11 refer to items to be used in a future assessment. Using the item sampling design as used in the current study, the number of items could be increased 5 per booklet without any modification in the administration of the tests. While it is clear that the use of common items reduced the amount of target information, the information gained by a common item, while not related specifically to the curriculum, enables one to get a total picture of mathematics learning. It provides a view of the development of mathematical concepts through the Divisions. Open-ended items, although more costly to grade, are essential for assessing basic facts and computation and can effectively be used in many other questions.

Recommendations 12 and 13 grow out of subjective judgements of the researchers. No assessment will work without full cooperation from classroom teachers. That cooperation can be secured through teachers having confidence in the study and knowledge about its operation in the broadest sense. The validation and administration procedures were designed to maximize these two aspects.

The variables named in Recommendation 14 provide not only information directly relevant to this assessment but they also make accessible, at very low cost, answers to other important research questions. It is also possible that depending on the "needs assessment" other background variables might be of interest in a future assessment.

PART II

RESULTS

Introduction

The analysis of individual item results which follows is organized by content:

Chapter II	Number
Chapter III	Algebra
Chapter IV	Geometry
Chapter V	Measurement
Chapter VI	Statistics
Chapter VII	Consumer Mathematics (grade 12 only)

Part II is completed with Chapter VIII, "Other Variables". Chapter VIII contains the analysis of the data relative to the sex variable, size of school variable and mathematics program (grade 12) variable. Also included in Chapter VIII is a summary of the Background Information obtained from the students.

For each of the Chapters II through VI a matrix of percentages is presented with thought level along one dimension and Division labels along the other. In these cluster analyses it is important to note that the analysis is done on targeted items for each Division so comparisons across Divisions is not possible.

Each cluster analysis is followed by a listing of the relevant subtopics that were used in choosing items.

This is followed by an analysis of the performances on the individual items in the pertinent content strand across all Divisions. Selected exercises are shown as illustrations.

The following notation, "a, b, c, d", is used to indicate that for a particular item the percentages of correct responses for Divisions I, II, III and IV are a, b, c, and d respectively. A variation occurs when the notation, "a, b, c, _", is used to indicate that an exercise was not given in Division IV, etc.

II Number

The overall results in number are given in the figure below.

	I	II	III	IV
Knowledge	76 (27)*	81 (17)	77 (3)	69 (4)
Comprehension	63 (21)	57 (6)	72 (9)	71 (2)
Application	54 (1)	49 (2)	56 (3)	67 (2)

Figure II-1 Cluster Analysis for Number

*(27) refers to the number of items in this category.

The total number of number items is 97. The analyses of the single items is reported under:

1. Natural Number
   - Numeration
   - Operation, Properties, Patterns
   - Basic Number Facts
   - Computation
2. Integers
3. Rational Numbers
   - Common Fractions
   - Decimal Fractions
   - Computation
4. Percent
5. Real Numbers

Examining the matrix above indicates a general trend for Application to be the most difficult. Comprehension shows up as the next most difficult except in Division IV where the Knowledge cluster was next; with the Comprehension questions being the easiest.

II.1 Natural Number

II.1.1 Numeration

Whole number numeration was assigned thirteen exercises dealing basically with two main ideas: (1) reading a numeral, and (2) place value and related concepts.

Reading a 3-digit numeral proved to be a simple task even for Division I. Over 91% of the grade 3's could recognize 251 as two hundred fifty one. Table II-1 shows the exercise and the percentage of each Division giving the correct response. The 98% performance by the Division II students shows two things: (1) the complete mastery of students in reading 3-digit numerals, and (2) that the students weren't just "goofing off" when they took the items.

Table II-1: Ex 102

251 is read as

A) two hundred fifty one
B) twenty five hundred one
C) one hundred fifty two
D) twenty five one
E) I don't know

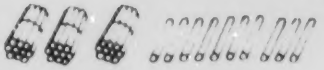
DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
* A	91.4	97.7		
B	3.0	0.4		
C	0.7	0.2		
D	1.9	1.4		
E	1.4	0.0		
OMIT	1.6	0.4		

Reading a 5-digit numeral turned out being only a little less convincing than the 3-digit case. It would appear that by the time students complete grade six, they are capable of reading numbers. Do they understand what they are reading? Ten items were designed to assess whether students understood the symbols used to express numbers. Basic to this understanding are the notions of grouping and of place value. By grade 3, 70% of the students comprehend the grouping by tens idea as assessed by exercise 108 shown in Table II-2.

Table II-2: Ex 108

Which picture below shows the same number as in the box?

A)
B)
C) 
D)
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
* A	69.7			
B	2.7			
C	5.7			
D	8.4			
E	12.6			
OMIT	0.9			

Connecting bundles of sticks with place value notation turned out to be a slightly more difficult task with 65% of the grade 3's able to select the correct numeral to go with a pictorial display of bundles of sticks.

The remaining nine exercises dealt with some aspect of place value. Performance at the grade 3 level ranged from a low of 27% to a high of 93%. These extreme items are shown in Tables II-3 and II-4 respectively.

Table II-3: Ex 104

What number is 3 more than 999?

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
RIGHT	26.8	65.6		
WRONG	72.5	34.2		
OMIT	0.7	0.2		

Table II-4: Ex 107

Which number is the largest?

A) 1234
B) 2341
C) 3412
D) 4123
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	1.5			
B	2.6			
C	1.6			
* D	92.7			
E	1.3			
OMIT	0.4			

It would appear that grade three students find it much more difficult writing the number that is 3 greater than 999, than they do in comparing the value of any set of 4-digit numerals.

On tasks relating to place value such as rounding to the nearest ten, students showed increasing competence over the grades reaching 96% at Division III. Even when exercises were couched in algebraic form, students performed at high levels, as shown in Table II-5.

Table II-5: Ex 203

If 583 = 500 + n + 3, then n =

A) 8
B) 80
C) 83
D) 800
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		11.2	2.1	
* B		78.1	94.3	
C		6.1	2.3	
D		2.0	0.6	
E		2.5	0.0	
OMIT		0.2	0.8	

In summary, the results indicate a relatively high and early capability in reading numerals (80% at Division I). This is accompanied by an early understanding of place value notions (65% at Division I) rising steadily and convincingly to over 90% at Division III.

II.1.2 Operations, Properties, and Patterns

Perhaps the most basic aspect of primary school mathematics, in Alberta at least, is the cardinal number property of a set of objects. Mastery of this notion is very strong as suggested by exercise 114 shown in Table II-6.

Table II-6: Ex 114

How many eggs are in the box?

A) 6
B) 7
C) 8
D) 9
E) I don't know

DISTRIBUTION OF RESPONSES

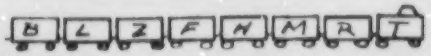
RESPONSE	DIVISION I	II	III	IV
A	0.6			
* B	98.2			
C	0.6			
D	0.2			
E	0.4			
OMIT	0.2			

Over 98% of the Division I students could identify the number of eggs in a carton of eggs partially filled. Identifying the ordinal number of a car within a train gave lower performance levels for Division I, but the percentage of success rose quickly (90% at grade 6 and 96% at grade 9). Table II-7 shows the relevant exercise.

Table II-7: Ex 116

If car B is the 15th car in a freight train and car L is the 16th, which car is the 20th?

A) car Z
B) car T
C) car H
D) car M
E) I don't know



DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	10.9	3.6	1.1	
B	18.8	3.4	1.1	
C	4.0	1.2	1.1	
* D	54.0	90.5	96.2	
E	10.9	1.4	0.4	
OMIT	1.4	0.0	0.0	

It would appear then that basic use of cardinal and ordinal number are accomplished by the end of grade 6.

As with the cardinal and ordinal number exercises, Division I students also showed 75% success with items dealing with the commutative property and the additive inverse. Typical is the performance on exercise 112 dealing with the commutative property as shown in Table II-8.

Table II-8: Ex 112

Which is the missing number?

$$24 + 76 = 76 + \square$$

A) 24
B) 34
C) 52
D) 100
E) I don't know

DISTRIBUTION OF RESPONSES

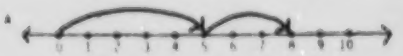
RESPONSE	DIVISION I	II	III	IV
* A	79.2			
B	1.1			
C	3.2			
D	8.7			
E	6.2			
OMIT	1.8			

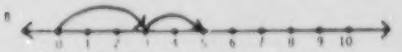
Basic understanding of simple sequences of numbers was also relatively strong. Filling in blanks in sequences such as 5, 8, 11, 14, __, or 5, __, 11, 14 resulted in levels of performance of 80% for Division I rising to 93% and 97% by Divisions III and IV.

Understanding of the four operations (+, -, x, ÷) was not as strong as that exhibited on the other items in this section. Somewhat typical was the response to an item dealing with subtraction on the number line. Only 20% of Division I students could identify the number line showing 5 - 3. Even at grade 12, the percentage was only 66%. This exercise is shown in Table II-9 because one of the alternative answers played a key role.

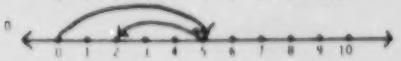
Table II-9: Ex 121

Which numberline shows 5 - 3?

A) 

B) 

C)

D) 

E) I don't know

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
A	7.4	2.8	0.6	0.9
B	14.9	7.2	1.5	1.9
C	53.9	63.7	38.7	29.9
* D	20.0	25.6	57.5	65.9
E	2.5	0.4	0.8	1.2
OMIT	1.4	0.4	1.0	0.2

An examination of alternative C in exercise 121 shows how strong a distractor it was: the percentage choosing it began at 54% at Division I, decreasing steadily to 30% at Division IV. Alternative C is the number line showing 5 - 2. Its ability to entice the students might in part be due to hastiness of response since it is difficult to believe that 30% of the Division IV sample cannot distinguish 5 - 3 from 5 - 2 on the number line. In contrast to exercise 121 was exercise 119, which showed skip counting by fives on the number line (a prelude to multiplication). Division I students gave an 82% performance on this item, compared to 20% for the subtraction exercise. Interpreting multiplication in terms of sets turned out to be intermediate in terms of difficulty with 40% of Division I giving the correct response.

An interesting consumer application item involving all four operations is shown in Table II-10. It indicates that 60% of the grade 6 students and 81% of the grade 9 students are able to intelligently cope with "sale prices" as compared with "regular prices".

Table II-10: Ex 206

A store is selling 5 cans of corn for 89¢. If the regular price is 19¢ per can, how much is saved by buying 5 cans at the sale price?

A) 70¢
B) 14¢
C) 6¢
D) 5¢
E) I don't know

DISTRIBUTION OF RESPONSES

	DIVISION		
RESPONSE	I	II	III	IV
A		16.4	5.7	
B		14.2	5.7	
* C		59.5	80.9	
D		7.0	4.1	
E		2.4	3.3	
OMIT		0.6	0.4	

Exercise 206, perhaps more than any other in this section, indicates that by grade 9, over 80% of the students are able to apply their number understanding in a real world situation.

II.1.3 Basic Number Facts

Fundamental in any mathematics program is the mastery of the basic number facts such as 3 X 6, 12 - 7, 3 X 4, 24 ÷ 8 and so on. Table II-11 shows several of these number facts and the performance of the students.

Table II-11
Ex 123, 124, 125, 126, 133, 134, 207, 209, 211

Ex No	Fact	I	II	III	IV
125	3 + 0 = ___	95			
123	7 + 6 = ___	94	98		
124	8 + 9 = ___	92			
126	3 - 0 = ___	84			
133	13 - 6 = ___	90	99		
134	3 X 0 = ___	89			
209	6 X 9 = ___	60	96		
207	54 ÷ 9 = ___		94		
211	24 ÷ 3 = ___	71	97		

It should be noted that all of the exercises dealing with number facts were open ended, i.e. the student had to fill in the answer, not choose from a set of answers. The contractors believed that generating the correct answer rather than recognizing it was a more appropriate context for assessing knowledge of basic facts.

The most noteworthy aspect of the percentages shown in Table II-11 is their high magnitude. In fact, the median value for all the addition and subtraction basic facts at Division I was 94%. For Division II it was 98%. Most of the multiplication and division facts are targeted at Division II. Nevertheless, the median Division I response on these items was 80%. Division II scored 96%. It seems safe to conclude that by the time a child leaves the elementary school, in all probability he has mastered the basic number facts.

Of some interest are the so called "zero" facts which some children have difficulty with. However, as shown in Table II-11, about 90% of grade 3 pupils know the zero facts.

II.1.4 Computation

As with the basic number facts, all computation exercises were open ended in format. A total of fourteen computation exercises using whole numbers was included in the assessment. Of these, six are shown in Table II-12.

Table II-12
Ex 135, 140, 137, 141, 214, 215

Ex No	Item	I	II	III	IV
135	38 + 19	79	92		
140	405 + 306	82	96		
137	36 - 19	60	93	92	
141	500 - 49	37	85		
214	43 X 9	31	90	94	
215	714 ÷ 6	3	82	89	

It should be noted that all computation exercises were arranged vertically in the exercise booklets.

Addition seems to be the easiest of the four algorithms for Division I students. Approximately 80% of the grade 3 students could add 2 and 3 digit numbers correctly. For Division II, the median percentage on addition exercises was 93. Subtraction was more difficult for both Division I and II; the median percentages being 48% and 86% respectively. Computing with zeroes in the minuend (as in Exercise 141) proved difficult for Division I, but by Division II the percentage had risen to 85.

The multiplication and division algorithms turned out to be rather difficult for Division I students but this is not surprising since the bulk of the instruction on these skills occurs in Division II. At Division II the performance levels reached 90% for multiplication and 82% for division. These results are somewhat comparable to, although a little lower than, the subtraction results.

At grade 9, the median response on the four exercises given, one for each operation, was 93% with a low of 89% on a division exercise and a high of 96% on an addition exercise.

No computation exercises were given at grade 12. In retrospect, the decision not to give any such exercises at the Division IV level seems to have been a good one since the grade 9 results were high.

11.2 Integers

Four items were assigned to the topic of integers. Of these, two were targeted at the Division III level and the other two at Division IV.

Table 11-13 shows an application exercise involving the subtraction of negative numbers. The performance levels of 58% and 77% at Division III and IV respectively are very much below performance levels on similar problems dealing with natural numbers. Likewise, the performance on simple computation exercises involving signed numbers (Table 11-14) is similarly much below comparable subtraction exercises involving only natural numbers.

Table 11-13: Ex 305

The air temperature on the ground is 31 degrees. On top of a nearby mountain, the temperature is -7 degrees. How many degrees difference is there between these two temperatures?

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT	9.3	21.4	58.2	77.2
WRONG	89.2	78.0	41.4	22.2
OMIT	1.6	0.6	0.4	0.6

Table 11-14: Ex 404

Subtract

(11) - (-13) =

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT			52.8	73.6
WRONG			46.7	26.2
OMIT			0.6	0.2

Note the performance of the grade 6 students on exercise 305 (Table 11-13). Over 20% of them could correctly solve this problem involving subtraction of a negative number even though formal instruction on this topic is scheduled later in the mathematics curriculum.

In all fairness to the grade 12 students who are academically oriented, it should be stressed that for students who had or were studying Mathematics 30 the results were much better. Their median response on the integer items was 88%. However, even this figure is below the overall grade 9 figure on exercises dealing with whole number computation (92%) although the difference (4%) could be attributed to sampling error.

11.3 Rational Numbers

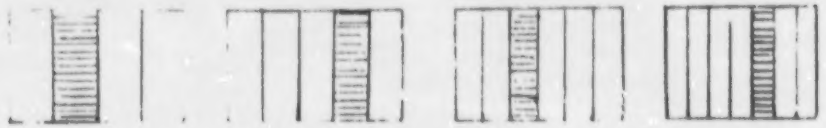
The rational number exercises dealt with three broad areas: (1) common fractions, (2) decimal fractions, and (3) decimal fraction computation. Overall it turned out that students did best on the computation, followed by common fractions and then decimal fractions. In general, levels of performance on the rational number items was considerably lower than on the natural number exercises.

11.3.1 Common Fractions

Levels of performance on basic concept of fraction items varied a great deal. The best performance occurred on exercise 144 shown in Table 11-15, in which students had to identify 1/5 as shown by vertically partitioned rectangles.

Table II-15: Ex 144

Which box is one-fifth ($\frac{1}{5}$) shaded?

A)
B)
C)
D) 

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	5.8	1.8		
* B	62.4	89.9		
C	10.2	4.3		
D	13.1	3.6		
E	7.7	0.4		
OMIT	0.7	0.0		

Performance fell by 20% when a mixture of shapes was used. The 90% performance by Division II students on exercise 144 was the highest level they reached on any of the 28 items dealing with the rational number area. At the other extreme, one of the lowest levels of performance on a targeted item occurred at Division III on a seemingly simple exercise that gave the number of boys and girls in a classroom and asked for the fraction of the class that was boys. Only 38% of grade 9 students gave the correct answer.

Another example of a seemingly simple exercise is shown in Table II-16. Although it would seem that the task of finding n in 2/7 + n = 1 would be quite simple for grade 9 students, less than one half could do so. Note that nearly as many grade 6 students could give the correct answer (45% vs. 48%).

Table II-16: Ex 306

What is n ?

$\frac{2}{7} + n = 1$

A) $\frac{7}{2}$

B) $\frac{5}{7}$

C) 5

D) $\frac{9}{14}$

E) I don't know

DISTRIBUTION OF RESPONSES

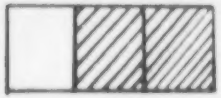
RESPONSE	DIVISION I	II	III	IV
A	30.4	34.1	44.4	
* B	15.3	44.6	47.9	
C	5.1	7.8	2.4	
D	7.0	5.5	0.6	
E	38.1	7.8	4.1	
OMIT	4.2	0.2	0.7	

An item dealing with equivalent fractions fared much better. The item is shown in Table II-17. Note that Divisions II, III, and IV all performed at approximately the same level.

Table II-17: Ex 307

The shaded part of which figure represents the same fraction as

A)

B) 

C)

D)

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	8.0	5.5	6.3	7.4
* B	34.9	84.9	86.4	86.3
C	31.5	2.4	2.1	2.9
D	4.9	6.3	3.8	2.3
E	15.7	0.6	0.0	0.9
OMIT	5.1	0.4	1.3	0.1

Lest the inference be made that on common fractions, Division III did only about as well as Division II, it should be noted that on two other items, one dealing with the multiplicative inverse, and the other with identifying a fraction on a number line, Division III performed at about the 80% level and Division II at the 50% level. On these two items, Division IV performed at the 90% and 86% levels respectively while those with Mathematics 30 reached 98% and 95% respectively.

II.3.2 Decimal Fractions

The new (1977) Curriculum Guide for elementary school mathematics places the beginning work with decimal fractions at an earlier level than before. It is thus not surprising that performances of grade 3 students on items targeted at Division I would be relatively low. In fact, the median response of grade 3 on the four decimal fraction items targeted at Division I was 12%. Typical of this performance is the percentage of correct responses on exercise 146 shown in Table II-18.

Table II-18: Ex 146

6 tenths =

A) 6.0
B) 0.6
C) 0.06
D) 0.006
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	58.3			
* B	13.1			
C	10.4			
D	3.4			
E	12.9			
OMIT	1.9			

This exercise is a Knowledge item asking only for the reading of a simple one place decimal fraction. That only 12% could respond correctly (and this in a multiple choice situation) suggests that something is misplaced or that instruction was absent.

Division II students did quite a bit better (72%) on similar Knowledge items perhaps reflective of the fact that such items in the past would more properly be targeted at Division II. It remains to be seen whether proper instruction can bring grade 3 students to respectable levels of performance. Other Knowledge items asking for decimal fraction equivalents for simple unit fractions gave rise to low levels of performance at grade 6, for example 1/5 (27%). Slightly more complex items dealing with equivalence gave rise to similarly low performances in grade 6 and significant improvement in performance did not occur at grade 9 or 12. Typical of such performances are the results on exercise 402 in Table II-19.

Table II-19: Ex 402

Three of the four numerals below name the same number. Which numeral names a different number?

A) 0.05
B) $\frac{5}{1000}$
C) 50% of 0.1
D) $\frac{1}{20}$
E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	I	DIVISION II	III	IV
A		5.0	2.8	3.3
* B		24.6	42.9	66.7
C		14.2	14.7	9.4
D		49.1	36.5	17.9
E		6.8	2.6	2.8
OMIT		0.4	0.4	0.0

In general, performances at Divisions III and IV did not top out, the median responses on all decimal fraction items (except computation) being 56% and 72% respectively. These compare with 12% and 27% at Divisions I and II respectively. Typical of Division III and IV performance are the results of exercise 403 dealing with exponential notation shown in Table II-20.

Table II-20: Ex 403

0.0032 is equal to

A) 32×10^{-4}

B) 32×10^{-3}

C) 32×10^{-2}

D) 32×10^{-1}

E) I don't know.

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
* A			54.4	74.7
B			18.6	11.4
C			18.6	8.8
D			3.6	2.6
E			3.6	2.5
OMIT			1.2	0.1

II.3.3 Computation (Decimal fraction)

Although it is somewhat anomalous that people can compute correctly with quantities they don't really understand, this seems to be the case with decimal fractions: the results for this section are considerably higher than those for section II.3.2. Results for exercises 223, 310, 311, and 312 are typical and all are shown in Table II-21.

Table II-21: Ex 223, 310, 311, 312

Ex No	Item	I	II	III	IV
223	6.15 + 7.38		85	95	
310	52.36 - 41.5		78	85	
311	43.23 x 0.9		62	80	84
312	8.88 ÷ 3.7		52	65	

Overall, the median responses at Divisions II, III, and IV were 70%, 82% and 84%. These are considerably higher than the comparable median responses for section II.3.2 which were 27%, 56% and 72%.

As might be expected, the order of performance on the four operations was addition first followed by subtraction, then multiplication, and then division.

II.4 Percent

Three items were devoted to percent. Overall, performances on these items were approximately 10% lower than those dealing with decimal fraction computation. The best performance was obtained on exercise 314, shown in Table II-22. These performances are between 10-15% higher than on the other two items.

Table II-22: Ex 314

8 is 25% of what number?

A) 2
B) 16
C) 32
D) 200
E) I don't know

DISTRIBUTION OF RESPONSES

	DIVISION		
RESPONSE I	II	III	IV
A	5.1	2.7	4.9
B	11.4	3.4	1.5
* C	61.3	84.2	84.8
D	14.0	8.4	7.3
E	8.1	1.0	1.4
OMIT	0.0	0.4	0.1

82

The grade twelve performance (85%) is no better than the grade nine performance (84%), a pattern that is somewhat typical on simple items that require practice for maintenance of skill.

11.5 Real Numbers

Only one item was given on this topic. It asked the student to identify which number from a set of four numbers was not a real number. The correct answer was the root of negative three. The item was given only at grade twelve and 68% gave the correct identification. The result for the Mathematics 30 people was 88%.

III Algebra

The overall results in algebra are given in the figure below.

	I	II	III	IV
Knowledge	__ (0)	__ (0)	53 (11)	53 (7)
Comprehension	68 (5)	42 (1)	51 (5)	51 (12)
Application	68 (4)	66 (6)	44 (5)	56 (8)

Figure III-1 Cluster Analysis for Algebra

The total number of algebra items is 64. The analysis of the single items is reported under the following topics:

1. Linear Expression,
2. Linear Equations and Inequalities,
3. Linear Functions,
4. Quadratics,
5. Relations and Variations,
6. Coordinate Geometry,
7. Exponents and Logarithms,
8. Polynomials,
9. Rational Relationships, and
10. Other Algebraic Topics

The formal algebra curriculum begins in Division III and becomes very strong in certain programs in Division IV. All of the algebra items in Division I and II are in the linear equations and inequalities topic as the other algebra topics become quite technical and specialized. There were no Knowledge items targeted in these Divisions.

Examining the matrix indicates no general trend for the difficulty of clusters at Divisions II and IV. Application questions were the easiest. It is incorrect to generalize that Division II is the poorest in Algebra. Relative strengths of Divisions show up more clearly in common information rather than target information.

III.1 Linear Expression

Of the five items in this sub-topic, four are given at Division III and one at Division IV. One of the algebraic manipulation items produced the following results.

Table III-1: Ex 316

$r + s - (r - s) =$

A) 0
B) 2r + 2s
C) 2r
D) 2s
E) I don't know

DISTRIBUTION OF RESPONSES

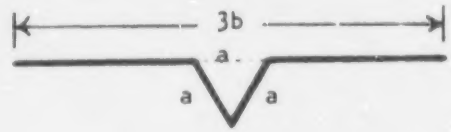
	DIVISION			
RESPONSE	I	II	III	IV
A			41.8	16.5
B			19.0	12.5
C			4.4	4.7
* D			28.7	61.5
E			5.0	4.6
OMIT			1.2	0.2

A similar item but with only one variable had a percentage correct of 55%. An application item in this category is shown in Table III-2 below.

Table III-2: Ex 319

Find the length of the heavy line.

A) 3b + 2a
B) 3b - a
C) 3b - 2a
D) 3b + a
E) I don't know



DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
A			56.0	46.0
B			5.3	5.2
C			5.1	5.6
* D			29.3	39.9
E			3.8	3.4
OMIT			0.4	0.0

It indicates a rather small difference between the two divisions with Mathematics 30 recording a 54%. Another item also requiring a variable for an answer came in rather low at __, __, 41, 72. This would suggest students having difficulty when the answer is abstract rather than having a concrete goal.

The one item targeted at Division IV asking for a verbal description of a linear expression had a performance of __, __, 57, 67.

III.2 Linear Equations and Inequalities

A total of 23 items were used in this category, nine at Division I, 7 at Division II, 5 at Division III, and 2 at Division IV.

Division I. Typical of the inequality items at Division I was Ex 151 below.

Table III-3: Ex 151

Which statement is true?

A) $3 + 5 < 4 + 4$
B) $3 + 4 > 5 + 3$
C) $3 + 5 > 4 + 3$
D) $5 + 4 < 4 + 3$
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	33.6	13.2	6.0	2.7
B	8.0	6.0	3.6	1.5
* C	33.8	71.7	85.7	93.8
D	7.7	7.6	4.0	1.6
E	14.1	1.4	0.8	0.2
OMIT	2.9	0.2	0.0	0.1

Application items in this category are represented by Ex 154 shown below.

Table III-4: Ex 154

Donna has 6 puppies. Four are brown and the rest are white. How many are white?

A) 1
B) 2
C) 3
D) 4
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	1.1			
* B	89.4			
C	1.6			
D	2.7			
E	2.1			
OMIT	3.2			

Another similar application question but involving adding and multiplying showed 71%.

A third type of question using a box "□" in a simple mathematical equation and asking the student to find the value of the box showed a performance of 83, 95, 97, __. However, when the question was asked in the form of an "if - then" statement such as in Ex 157 below, the performance was lower, suggesting that the if - then statement is slightly more difficult for the grade 3 student.

Table III-5: Ex 157

If 3 + □ = 10, then □ =

A) 13
B) 7
C) 10
D) 30
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	5.3			
* B	74.8			
C	3.3			
D	1.6			
E	10.2			
OMIT	4.7			

Division II. All but one of the seven items in this category were application. The one comprehension question (Ex 226) shows interesting growth from one Division to the next.

Table III-6: Ex 226

If x is less than 4, then x + 7 must be

A) less than 7
B) less than 11
C) greater than 7
D) greater than 11
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		16.2	12.6	4.2
* B		42.2	69.5	85.4
C		32.0	13.6	8.4
D		6.9	1.5	0.9
E		2.4	1.5	1.1
OMIT		0.4	1.2	0.0

The following are representative of the application items.

Table III-7: Ex 229

Marie took four spelling tests. Each test had 30 words. On the four tests she spelled correctly the following numbers of words:

25, 23, 27, 24.

Altogether, how many words did she spell incorrectly on all four tests?

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT		62.2	77.5	87.1
WRONG		37.3	22.1	12.5
OMIT		0.6	0.4	0.3

Table III-8: Ex 230

Sam has 51 pop bottles and 8 cartons. Each carton holds 6 bottles. If Sam fills all the cartons, how many bottles will be left over?

A) 6
B) 8
C) 3
D) 14
E) I don't know

DISTRIBUTION OF RESPONSES

		DIVISION			
	RESPONSE	I	II	III	IV
	A	8.1	6.4	3.1	1.1
	B	11.9	6.4	1.7	1.4
*	C	33.0	77.6	92.4	96.0
	D	23.9	5.6	1.5	1.1
	E	19.0	3.6	1.1	0.2
	OMIT	4.2	0.4	0.2	0.2

The latter tops out at Division III while the former does so at Division IV. All the Division II items were verbal problem types and perhaps the relatively low scores reflects a reading factor.

Division III. The three Knowledge items in linear equations and inequalities asked students to find the value of x. Ex 320 below is typical.

Table III-9: Ex 320

Find x if

$3x - 3 = 12$

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT		54.2	79.2	88.1
WRONG		45.6	19.4	11.7
OMIT		0.2	1.5	0.1

This item shows an interesting growth pattern across divisions while another involving a negative value of x came in at .7, 1.4, 32, 71 across the four divisions. This indicates that Division I and II have very little ability to deal with negative numbers in an algebraic situation. In fact, Division III scored only 32%.

Two application items differed considerably in difficulty. A typical verbal type, Ex 323 below, was very well answered.

Table III-10: Ex 323

John is four years older than Ellen, and Ellen is 11 years younger than Monica. Monica is 12 years old. How old is John?

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT			85.1	91.5
WRONG			13.8	8.2
OMIT			0.7	0.3

The second more difficult application item involved finding the sides of a rectangle given some information on the relationship between its sides. The distractors for the answer were subtle and it resulted in a relatively low __, __, 23, 42. This may suggest a weakness in solving problems algebraically, but it also suggests that multiple choice questions can be made more difficult with appropriate distractors.

Division IV. Two applications items at this division were of equal difficulty. One is given here in Table III-11.

Table III-11: Ex 410

"Mary paid three times as much as Al for her shoes. If they spent 36 dollars altogether, how much did Al spend for his shoes?" What equation is related to this problem?

A) $3x = 36$

B) $3x + 3 = 36$

C) $3x + x = 36$

D) $x + x = 36$

E) Don't know.

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
A			29.0	19.9
B			4.4	3.0
* C			60.4	74.6
D			3.6	1.3
E			2.3	1.1
OMIT			0.4	0.1

The other item dealing with inequalities also involved translating an English statement to a mathematical expression. This reading-mathematical task then resulted in 60% and 75% performances at Divisions III and IV.

III.3 Linear Functions

All three items here were targeted at Division III. The items were of equal difficulty although they were all very different. Finding the value for x given a value for y in an equation like $5x + 6y = 24$ was answered by 56% of Division III. An item interpreting a formula is reported here.

Table III-12: Ex 326

Some people suggest that the following formula be used to determine the average mass for boys between the ages of 1 and 7

$$M = 9 + 3A$$

where M is the average mass in kilograms and A is the boy's age in years.

According to this formula, for each year older a boy gets, his mass should be

A) 12 kg more

B) 9 kg more

C) 3 kg less

D) 3 kg more

E) I don't know

DISTRIBUTION OF RESPONSES

		DIVISION		
RESPONSE	I	II	III	IV
A			26.2	0.7
B			12.8	21.9
C			2.6	2.3
* D			49.5	70.6
E			8.9	4.3
OMIT			0.0	0.1

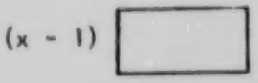
The third item involved discovering a linear equation given a set of ordered pairs came in at 54%. This means that linear function proficiency is attained by one-half of Division III students.

III.4 Quadratics

Of a total of five items on this topic, two were targeted at Division III; both of these were in the area of factoring a trinomial quadratic equation and finding the product of binomials. These had performances around 55%. The remaining three were Application items targeted at Division IV. A representative of this group is Ex 413 on the next page.

Table III-13: Ex 413

For what values of x is the area of the rectangle greater than 18?



A) All x such that $x > 7$ or $x < 4$.

B) All x such that $4 < x < 7$.

C) All x such that $-7 < x < -4$.

D) All x such that $x < -7$ or $x > -4$.

E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				19.7
* B				44.7
C				11.9
D				7.6
E				15.3
OMIT				0.8

The last item asked students to discover a simple quadratic physics formula given a set of ordered pairs. The performance was __, __, 58, 70.

III.5 Relations and Variations

The two items here were targeted at Division III and the performances show that the question which asked for a variable as the answer, such as t/25, showed decrease in performance from 81% to 27%. The easier item is shown below in Table III-14.

Table III-14: Ex 330

If $A = \ell w$ and if $A = 12$ and $\ell = 3$, then $w =$

A) $\frac{3}{4}$

B) 3

C) 4

D) 36

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		9.8	1.1	1.1
B		6.8	1.7	1.2
* C		48.3	80.6	90.2
D		21.8	14.1	6.2
E		12.4	2.3	1.4
OMIT		1.0	0.2	0.0

This shows interesting growth across divisions.

III.6 Coordinate Geometry

The first item here was targeted at Division III and asked the student to identify the graph of a simple linear equation. The responses were __, __, 41, 72. The three other items, targeted at Division IV, dealt with Knowledge and Comprehension of the coordinate plane.

One of the items required finding the slope of the graph of $2x - y = 5$, while another asked for the equation of a line marked on a graph parallel to the x axis and a given distance from it. The performance on these two items was 23 and 42 percent respectively. The final item on the quadrants of the coordinate plane scored 36%. One should interpret these results with the realization that Math 15 and 25 students, for example, have very little experience in this area, as is the case in many areas following in this algebra section.

III.7 Exponents and Logarithms

Of four items given in this category, two were targeted at Division III and two at Division IV. The results for evaluating a power are shown in Table III-15 below.

Table III-15: Ex 333

What is 4^3?

A) 7
B) 12
C) 64
D) 81
E) I don't know

DISTRIBUTION OF RESPONSES

	DIVISION		
RESPONSE I	II	III	IV
A.	9.1	0.9	
B	44.8	7.2	
* C	36.1	89.3	
D	3.0	1.5	
E	6.5	1.1	
OMIT	0.6	0.0	

The other item at Division III showed that about half of the students could identify an approximate square root of a number under 20.

One of the Division IV items indicated a low performance (19%) on identifying the graph of an exponential function. A final Comprehension question in this category shows how students reacted to the properties of powers.

Table III-16: Ex 419

Which of the following equations is false?

A) $3^4 \times 2^2 = (3^2 \times 2)^2$

B) $(-3)^3 = -3^3$

C) $2^2 + 3^2 = (2 + 3)^2$

D) $(-3)^4 = 3^4$

E) I don't know.

DISTRIBUTION OF RESPONSES

	DIVISION	
RESPONSE	III	IV
A	24.2	17.7
B	13.0	11.5
* C	22.9	54.5
D	34.7	9.9
E	5.0	6.1
OMIT	0.4	0.4

The reader might agree that the distractors make this item somewhat more difficult than the content alone does.

III.8 Polynomials

Seven items targeted at Division IV dealt with this topic. Three dealing with polynomial expressions were also given at Division III. Evaluating expressions with more than one variable was answered __, __, 65, 83. Another item on the division of one polynomial by a single negative term came out at __, __, 30, 58, while the third item reported below did not show as much difference between divisions.

Table III-17: Ex 422

Which formula represents the number of cents in q quarters and d dimes?

A) 25q + 10d

B) 35(q + d)

C) q + d

D) qd

E) I don't know.

DISTRIBUTION OF RESPONSES

	DIVISION	
RESPONSE	III	IV
* A	64.0	70.3
B	7.7	6.2
C	20.2	16.5
D	2.8	2.8
E	5.3	4.0
OMIT	0.0	0.2

The remaining four items all at Division IV, showed a range of values from 40 to 63. The two items on the next page indicate the type of response.

Table III-18: Ex 425

If $f(x) = 4x + 3$ and $g(x) = x^2 - 2$ then $f(g(x))$ equals

A) $x^2 + 4x + 1$

B) $4x^2 - 8$

C) $16x^2 + 24x + 7$

D) $4x^2 - 5$

E) I don't know.

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
A				23.0
B				5.0
C				10.7
* D				40.0
E				20.9
OMIT				0.4

Table III-19: Ex 426

If $P(x) = (x + 1)(x - 5)$ then then which of the following statements is true?

A) $P(6) < 0$

B) $P(20) > 0$

C) $P(0) = 0$

D) $P(-2) < 0$

E) I don't know.

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
A				11.6
* B				54.5
C				7.6
D				7.5
E				17.9
OMIT				1.1

The group of students with only one mathematics course scored only 14% on these two items. They have not had any experience with composite functions or quadratic inequalities.

III.9 Rational Relationships

The following item shows growth in the ability to use abstract symbolism.

Table III-20: Ex 428

Suppose $\frac{x}{y}$ represents a number. If the values of x and y are each doubled, the new number is

A) $\frac{1}{2}$ as large as $\frac{x}{y}$

B) equal to $\frac{x}{y}$

C) double $\frac{x}{y}$

D) 4 times as large as $\frac{x}{y}$

E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		7.6	4.7	5.6
* B		8.8	16.4	54.5
C		67.9	72.7	35.7
D		10.4	3.8	2.6
E		4.9	2.3	1.6
OMIT		0.4	0.2	0.0

Obviously, Division II and even Division III students were confused by the abstractness of the item.

Two items on solving a rational equation were given, one each targeted at Divisions III and IV. The Division III item is shown below.

Table III-21: Ex 335

If $\frac{5}{2n} - \frac{1}{2n} = \frac{1}{4}$, then n =

A) 8

B) 4

C) 2

D) $\frac{1}{2}$

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
* A			27.4	59.6
B			12.0	7.1
C			31.1	12.8
D			12.6	10.4
E			17.0	9.8
OMIT			0.0	0.2

This item shows substantial growth between divisions. The other item showed that half of Division IV students could solve symbolically for a given variable in a simple direct relation physics formula. The last item in this category asked to evaluate a rational function as the dependent variable increased to infinity. This indicated a performance of 35%.

III.10 Other Algebraic Topics

Two questions on systems of equations were handled by Division IV students at 74% and 90%. These students responded to naming a graph of a conic at 42% and identifying a specific trigonometry identity at 24%. A sequence item is given in Table III-22 below.

Table III-22: Ex 435

What number do you think will be next?

1, 1, 2, 3, 5, 8, __

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT				50.1
WRONG				49.1
OMIT				0.8

This shows half of the Division IV group detecting a Fibonacci sequence. The final item reported here was one given at Division III which showed 69% of these students being able to interpret the information given in a Venn diagram.

IV Geometry

The results by thought level for each Division are summarized in Figure IV-1 below.

	I	II	III	IV
Knowledge	77 (3)*	41 (5)	77 (4)	73 (2)
Comprehension	54 (12)	66 (6)	61 (4)	50 (4)
Application	40 (5)	56 (3)	57 (3)	47 (4)

Figure IV-1 Cluster Analysis for Geometry

*The number 77 refers to the mean percentage of correct responses. The number 3 in parenthesis indicates the number of Knowledge exercises targeted at Division I.

The geometry section was subdivided into the following topics:

1. Points, Lines, Planes and Space,
2. Transformations,
3. Angles,
4. Triangles,
5. Circles,
6. Polygons,
7. Line Relationships,
8. Symmetry,
9. Congruence,
10. Similarity, and
11. Deductive Systems.

The patterns of percentages shown in Figure IV-1 are generally what one would expect. They decrease in going from Knowledge to Comprehension to Application. A notable exception is the set of Knowledge exercises at Division II. In general, these exercises required a knowledge of geometric terms. Apparently this geometric terminology is not well known.

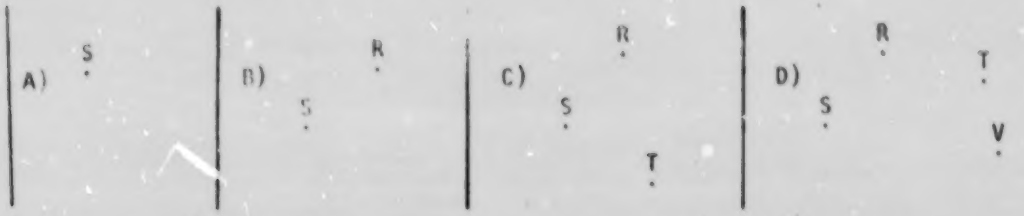
The low percentage at Division I in Application seems to reflect a low performance in working with three-dimensional figures.

IV.1 Points, Lines, Planes and Space

The study of geometry requires a basic knowledge of points, lines, and planes as they occur in two and three-dimensional spaces. It would seem that students do quite well with these parts of geometry in two dimensions. A typical example is Ex 436 in Table IV-1 on the next page.

Table IV-1: Ex 436

In which of the following can one and only one line be drawn through all of the given points?



E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		5.3	8.2	7.0
* B		80.3	83.1	88.1
C		5.3	2.7	1.1
D		7.5	3.1	1.9
E		1.4	2.7	1.3
OMIT		0.2	0.4	0.7

A number of exercises in this section related to three-dimensional figures. Although we live in a three-dimensional world, geometric knowledge of it is not high apparently. The exercises in Table IV-2 below are indicative.

Table IV-2: Ex 160 and Ex 162

Which one of the following has a shape MOST like an orange?

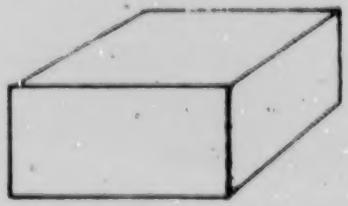
A) cone
B) cube
C) cylinder
D) sphere
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	12.2			
B	9.7			
C	26.0			
* D	35.3			
E	12.5			
OMIT	4.2			

How many corners does a box have?

A) 4
B) 6
C) 7
D) 8
E) I don't know



DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	43.6			
B	2.4			
C	6.6			
* D	44.9			
E	1.1			
OMIT	1.5			

This weakness in three-dimensional geometry is apparent in the upper Divisions as well. This is seen in the performances on the two exercises in Table IV-3 below.

Table IV-3: Ex 437 and Ex 438

How many legs should be put on a table in order to be certain that the table would not rock?

A) 2
B) 3
C) 4
D) 5
E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION III	DIVISION IV
A	0.8	0.9
* B	13.8	38.8
C	77.2	47.3
D	6.2	10.9
E	2.1	1.9
OMIT	0.0	0.2

If the segment $\overline{PQ}$ is an edge of a rectangular solid, how many faces of this solid contain neither P nor Q ?

A) 1
B) 2
C) 3
D) 4
E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION III	DIVISION IV
A	24.0	21.8
* B	34.1	36.4
C	14.9	12.5
D	16.9	17.8
E	9.0	11.5
OMIT	1.2	0.1

For Ex 437, most tables that are encountered in real life do have four legs. That could account for the attractiveness of alternative (c).

The percentages for Ex 438 reveal a certain weakness in Application problems in three dimensions. The performance at Division IV is especially noteworthy.

As a final note, for many of the exercises in this section, students from large schools did not do as well as their small and medium size school counterparts.

IV.2 Transformations

The one Knowledge exercise used in this section is shown in Table IV-4 below.

Table IV-4: Ex 235

The triangles shown are related by a _____.

A) slide
B) turn
C) flip
D) flop
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		38.5		
B		20.7		
* C		33.1		
D		2.8		
E		4.9		
OMIT		0.0		

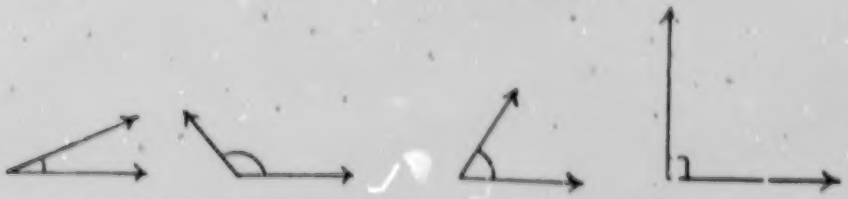
This result likely reflects the level of usage of transformations and motion geometry ideas in the elementary schools.

IV.3 Angles

In general, students did quite well in this section. The exercises shown in Table IV-5 below are indicative of growth at the upper Divisions.

Table IV-5: Ex 337 and Ex 339

Which of the angles below is the largest?



A) B) C) D)

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	2.2	3.3	2.3	
* B	8.4	61.1	85.6	
C	3.1	6.1	0.8	
D	83.2	28.8	9.2	
E	2.2	0.6	0.6	
OMIT	0.9	0.2	1.5	

Table IV-5 (cont.)

Estimate the number of degrees in angle Y of this triangle.

A) 60°
B) 90°
C) 30°
D) 120°
E) I don't know

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
* A	15.7	51.7	76.6	
B	7.8	13.6	3.5	
C	38.5	18.3	9.8	
D	2.5	11.6	8.1	
E	31.8	4.5	1.5	
OMIT	3.7	0.2	0.6	

One Comprehension exercise involving the construction of an angle bisector had a percentage of __, __, 58, __. This could be interpreted as a weakness in geometric construction. In another Comprehension exercise involving the complement and supplement of an angle, the pe centage was __, __, __, 37. These results could be attributed to forgetting previous macerial since they likely were not covered in grade 12.

IV.4 Triangles

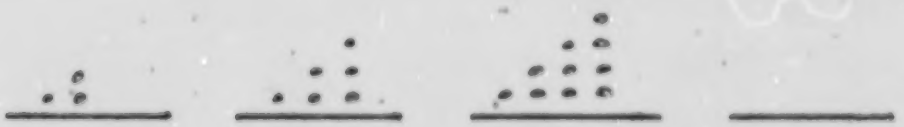
Triangles are a basic requisite in the study of geometry. Students seemingly learn to recognize triangles very early.

One of the exercises requiring the recognition of triangles had rercentages of 90, 97, __, __.

Students in all Divisions did quite well and showed steady growth on exercises related to triangular arrays. An exercise of this type is shown in Table IV-6 below.

Table IV-6: Ex 237

Draw the figure that would come next.



DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT	38.0	75.9	86.7	92.8
WRONG	60.0	23.6	12.9	6.8
OMIT	2.0	0.6	0.4	0.3

Exercises involving Applications of mathematics were troublesome throughout. This section was no exception, as shown in Table IV-7 below.

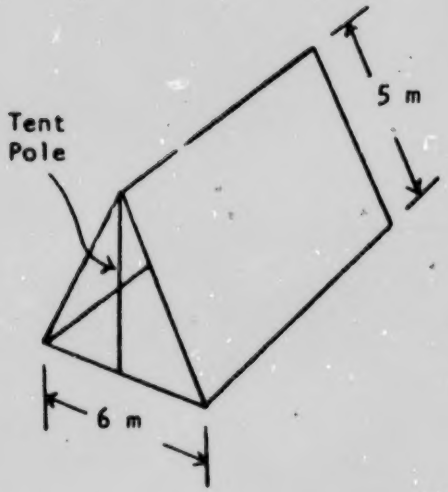
It is interesting to note that the percentages for girls and boys are 19 and 42 respectively for this exercise.

Another Application exercise, targeted at Division III, required the computation of the measure of one angle of a triangle given the other two angles. Its percentages were __, 30, 62, 80. Further, students with one high school mathematics course had a percentage less than 50.

Table IV-7: Ex 341

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT			30.4	
WRONG			68.7	
OMIT			0.9	



To set up a tent having the dimensions shown in the drawing, the vertical tent poles should be how many metres high?

IV.5 Circles

Circles are another fundamental part of the study of geometry. Generally students did well in this section. For instance, Division I students were 97 percent successful in recognizing circles. At the other end of the divisional spectrum we have Ex 440 shown in Table IV-8 on the next page.

Table IV-8: Ex 440

Point P is inside a circle with center O. Point S is nearer to O than to P. The location of point S is

A) on the circle.

B) outside the circle.

C) inside the circle.

D) not established with the given information.

E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A			3.6	2.0
B			3.2	1.5
* C			51.6	69.8
D			39.9	25.4
E			1.7	1.1
OMIT			0.0	0.2

Apparently the decoding task in this problem was difficult for both Divisions III and IV.

IV.6 Polygons

The items in this section were related to polygons of four or more sides. The recognition of rectangles, squares, etc., was relatively well done. One interesting item required the recognition of a square resting on one of its corners. Its percentages were 54, 66, __, __. Recognizing a square in its usual position would likely have given higher percentage values.

Recognizing a pattern of polygons was the focus of Ex 174 shown in Table IV-9 below.

Table IV-9: Ex 174

Which figure comes next?

A)

B)

C)

D)

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	20.6	25.4	8.4	
B	22.1	9.2	4.2	
C	10.6	5.9	1.5	
* D	32.1	58.1	82.6	
E	11.7	1.0	2.3	
OMIT	2.9	0.4	1.0	

The Division I and II students seemed to be influenced by misleading perceptual cues whereas the Division III student treated the exercise analytically.

An Application exercise is shown in Table IV-10 below.

Table IV-10: Ex 239

Shown below are two squares. X and Y are the centers of the squares. What is the distance in centimetres from X to Y?

A) 1 cm
B) 1.5 cm
C) 2 cm
D) 2.5 cm
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		9.2	5.1	
B		5.1	2.3	
* C		73.2	85.9	
D		9.0	4.4	
E		3.5	2.3	
OMIT		0.0	0.0	

In contrast, an Application exercise not done well is shown in Table IV-11 below.

Table IV-11: Ex 441

The largest possible square piece is cut from a rug that is 4 m by 6 m. How many square metres are there in the remaining piece?

A) 8
B) 2
C) 12
D) 24
E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
* A		11.6	29.0	42.1
B		38.9	35.4	24.6
C		11.4	7.1	7.8
D		28.2	20.0	16.5
E		9.6	8.2	8.8
OMIT		0.4	0.4	0.2

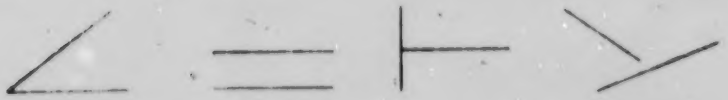
Apparently distractor (B) was popular because 2 m is the difference between 4 m and 6 m and distractor (D) was also popular because 24 m^2 is the product of 6 m and 4 m. Again, decoding verbal information seems to be a difficult task.

IV.7 Line Relationships

Two of the exercises used in this section are shown in Table IV-12 on the next page.

Table IV-12: Ex 240 and Ex 442

Which two line segments look perpendicular?

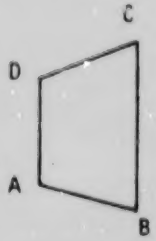


A) B) C) D) E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		7.1	12.0	
B		36.8	27.8	
* C		31.3	49.2	
D		16.1	4.1	
E		7.8	5.9	
OMIT		1.0	1.1	

In this figure a pair of parallel sides is



A) $\overline{AD}$ and $\overline{AB}$
B) $\overline{AD}$ and $\overline{BC}$
C) $\overline{AD}$ and $\overline{CD}$
D) $\overline{AB}$ and $\overline{CD}$
E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				0.2
* B				80.7
C				1.5
D				16.7
E				0.6
OMIT				0.2

From the distribution of responses for Ex 240, it is apparent that perpendicularity and parallelism are terms that are confused by many students.

Exercise 442 was done correctly by only 51% of the grade 12 students who have had only one mathematics course. As an aside comment, the distribution of responses would probably have been quite different if the parallel sides had been horizontal.

IV.8 Symmetry

This is a topic that probably does not receive much emphasis in the classroom. Symmetry and transformations are new to the geometry curriculum.

A typical exercise is shown in Table IV-13 on the next page.

Table IV-13: Ex 242

Which letter below does NOT have any line symmetry?

A) W
B) X
C) Y
D) Z
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		21.5	16.8	
B		14.5	9.4	
C		20.0	10.2	
* D		32.1	34.9	
E		11.6	28.0	
OMIT		0.4	0.7	

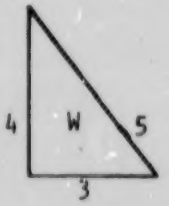
Performances on other similar exercises was about the same as this.

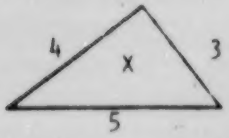
IV.9 Congruence

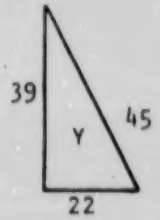
Congruence is a fundamental notion in the study of geometry. It seems to be well understood by students at all stages. A typical exercise is shown in Table IV-14 below.

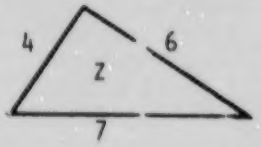
Table IV-14: Ex 345

Which of the triangles below is congruent to triangle W?









A) only X
B) only X and Y
C) only Y
D) only Z
E) I don't know

DISTRIBUTION OF RESPONSES

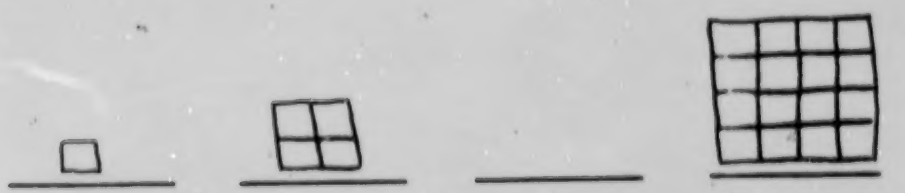
RESPONSE	DIVISION I	II	III	IV
* A		66.5	77.6	83.7
B		3.7	3.2	3.6
C		2.0	7.5	9.5
D		24.7	10.0	1.1
E		2.7	1.7	1.9
OMIT		0.4	0.0	0.2

IV.10 Similarity

This section included some exercises on pattern recognition based on similarity. A typical example is Ex 178 shown in Table IV-15 below.

Table IV-15: Ex 178

Draw the missing figure.



DISTRIBUTION OF RESPONSES

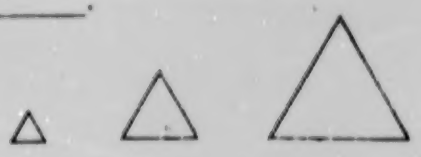
RESPONSE	DIVISION I	II	III	IV
RIGHT	25.1	41.2		
WRONG	71.9	58.7		
OMIT	2.8	0.2		

Recognition of similarity given only perceptual cues, seemed to be a difficult task. Note Ex 245 in Table IV-16 below.

Table IV-16: Ex 245

The figures below are _____.

A) congruent
B) similar
C) 3-dimensional
D) reflections
E) I don't know



DISTRIBUTION OF RESPONSES

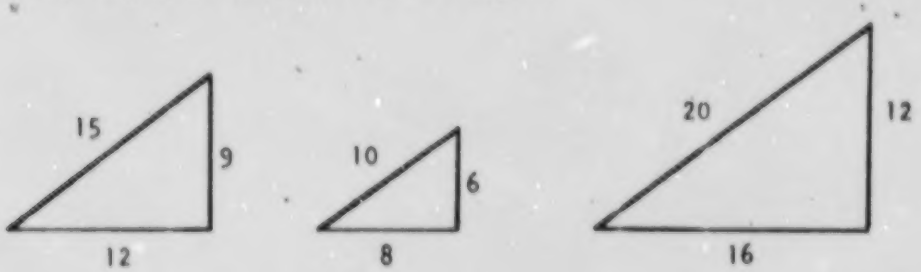
RESPONSE	I	DIVISION II	III	IV
A		13.0	14.7	20.8
* B		48.3	63.1	68.4
C		31.6	16.0	8.1
D		5.5	4.9	1.5
E		1.6	1.3	0.8
OMIT		0.0	0.0	0.4

The distribution of responses for Ex 245 is interesting in that grade 12 students did little better than grade 9 students. Furthermore, grade 12 students from Math 23 and 25 programs had percentages less than the grade 6 students.

Using the ratios of sides in similar figures seems to be a difficult task for many students. A good example is Ex 346 in Table IV-17 on the next page.

Table IV-17: Ex 346

The three triangles below have the same shape but not the same size. What is the ratio (in lowest terms) of the shortest to the longest side in each of the triangles?



A) $\frac{1}{2}$

B) $\frac{3}{5}$

C) $\frac{4}{5}$

D) The ratio changes for each triangle

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE		DIVISION I	II	III	IV
A			8.4	7.9	10.3
* B			13.9	34.3	56.0
C			10.2	4.8	7.1
D			57.7	44.2	23.3
E			9.6	7.6	2.8
OMIT			0.2	1.3	0.5

IV 11 Deductive Systems

Understanding deductive reasoning is a stated goal for the study of geometry at the high school level. It seems that this understanding is not obtained as indicated by Ex 445 shown in Table IV-18 on the next page.

Table IV-18: Ex 445

The following "proof" that $2 = 1$ is obviously not done correctly. Study the "proof" and circle the letter next to the step in which the mistake is made.

	1. suppose that	$y = b$
	2. multiply by y	$y^2 = by$
	3. subtract b^2	$y^2 - b^2 = by - b^2$
A)	4. factor	$(y - b)(y + b) = b(y - b)$
B)	5. divide by $(y - b)$	$y + b = b$
C)	6. substitution since $y = b$ (step 1)	$b + b = b$ or $2b = b$
D)	7. divide both sides by b	$2 = 1$
E)	I don't know.	

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				18.0
* B				21.7
C				31.5
D				10.9
E				17.5
OMIT				0.5

It is interesting to note how many students selected alternative (C). Apparently, because they knew that "2b = b" is false it was assumed that that must be where the mistake was made.

With most items, the Math 30 students performed at a much higher level than did the rest of the Division IV sample. However, for this item, the performance for the Math 20, the Math 23 and the students with one mathematics course was better than for the Math 30 students.

V Measurement

The overall results in measurement are given in the figure below.

	I	II	III	IV
Knowledge	57 (6)	72 (2)	88 (1)	65 (1)
Comprehension	(0)	61 (1)	68 (1)	47 (1)
Application	74 (4)	46 (3)	56 (3)	62 (5)

Figure V-1 Cluster Analysis for Measurement

The measurement strand in the assessment was assigned 28 exercises, ten of which were at the Knowledge level, three at the Comprehension level, and fifteen at the Application level. The emphasis on the Knowledge and Application levels was purposive on two counts: (1) with the coming of the metric system, familiarity with the system is quite variable; hence the emphasis on Knowledge items, (2) measurement is a process relating mathematics to reality, thus the emphasis on Application.

Only metric units were used in any item involving standard units. For this reason, some items may seem more difficult than they would a few years hence when metric measurement is not so novel.

The analysis of single items is reported under the following sub-topics:

1. Time and Money,
2. Length,
3. Perimeter and Area,
4. Volume and Capacity,
5. Mass and Temperature,
6. SI Relations.

The matrix above shows that only at Divisions II and III does the pattern of Application items being the most difficult hold.

V.1 Time and Money

Time and money are the only two properties reported upon in this survey which were not affected by a change in unit with the arrival of the metric system. For time, the basic unit is still the second, and for money the unit is still the dollar. On this basis, the expectation would be that students would do better on exercises involving time or money as compared to length, perimeter, mass, and so on and in general this turned out to be the case. In particular at the Division I level, 83% of the students could tell time looking at the face of a clock and over 78% could make change on a small purchase of less than 50¢. This compares very favourably with their performance on length items which averaged about 65%.

Representative of the level of performance on time items is exercise 247 shown in Table V-1 involving time duration. The 87% performance at grade 6 level suggests rather extensive mastery of concepts related to arrival and departure times.

Table V-1: Ex 247

On a certain day a train left Edmonton at 10:30 a.m. and arrived in Calgary three hours later. At what time did the train arrive in Calgary?

A) 7:30 a.m.
B) 12:30 p.m.
C) 1:30 p.m.
D) 2.30 p.m.
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		1.2	1.3	
B		8.6	2.9	
* C		86.7	92.8	
D		2.4	2.3	
E		1.0	0.6	
OMIT		0.2	0.2	

Exercise 181 involving coins was representative of the performance on money items and is reproduced in Table V-2 on the next page.

Table V-2: Ex 181

Jerry has 2 nickels, 1 quarter, and 4 pennies. How much money does he have in all?

A) 39 cents
B) 7 cents
C) 24 cents
D) 34 cents
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
* A	76.1	92.2		
B	6.8	2.5		
C	2.5	0.4		
D	8.1	4.1		
E	3.0	0.6		
OMIT	3.5	0.2		

V.2 Length

Three items dealt with the measurement of length, all three at the Knowledge level. The items essentially tested the students' knowledge of centimetres as a unit of length and whether they could estimate the length of simple objects in terms of that unit. Table V-3 shows exercise 186 in which the length of a crayon is correctly estimated by 72% of Division I students, rising to 90% for Division III.

Table V-3: Ex 186

About how long is this crayon?

A) 1 centimetre
B) 10 centimetres
C) 1 metre
D) 10 metres
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	5.5	0.8	1.5	
* B	72.6	88.6	90.4	
C	6.9	3.4	2.5	
D	6.9	6.2	1.7	
E	3.9	0.6	4.0	
OMIT	4.2	0.4	0.0	

Comparable performance was obtained on the other two length items.

V.3 Perimeter and Area

Nine items dealt with perimeter and/or area all at the Comprehension or Application level. None of the items were given at the Division I level.

At the simplest level, a question asking for the perimeter of a triangle with sides of specified magnitude was correctly answered by 61% of the grade 6 students. A similar question asking for the length of fencing required for a rectangular 9m x 5m garden, was done less well (32% of grade 6's; 58% of grade 9's). Even at the grade twelve level, the performance on perimeter items was not much higher. For example, the recognition of the formula involving perimeter as a function of length and width was done correctly by 73% of the grade 12's. This item is shown in Table V-4 on the next page.

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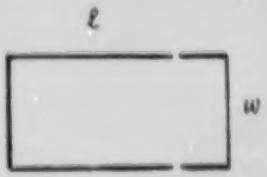
Table V-4: Ex 34P

The rectangular field pictured here has length ℓ and width w. The total distance around the edges of the field is given by the expression

A) ℓw
B) $2\ell w$
C) $\ell + w$
D) $2\ell + 2w$
E) I don't know



DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A			17.1	13.2
B			9.7	9.8
C			3.4	3.0
* D			67.8	73.3
E			1.5	0.4
OMIT			0.4	0.4

The 73% figure was the highest obtained by any grade on any of the nine items on perimeter and/or area. At the other extreme, the lowest percentages were obtained on an item involving the use of the relationships between area and perimeter of a square. Performance at the grade 9 level failed to exceed 35%. Table V-5 shows the item.

Table V-5: Ex 34Q

Mr. Johnson wants to buy carpeting for his living room. The room is square and has a perimeter of 28 m. What is the area of the room in m^2?

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
RIGHT		6.0	34.7	
WRONG		93.9	64.4	
OMIT		0.2	0.9	

Somewhat surprising was the performance on the item shown in Table V-6 below. Although it might be assumed that students would find it easier to recognize the formula for finding the perimeter of a rectangle given the length and width, apparently comparing areas of circles is no harder.

Table V-6: Ex 448

What fractional part of the interior of the large circle is shaded?

A) $\frac{1}{5}$

B) $\frac{1}{4}$

C) $\frac{1}{3}$

D) $\frac{1}{8}$

E) I don't know.

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
A			11.1	14.2
* B			71.4	71.9
C			8.5	5.9
D			5.7	4.3
E			2.4	3.4
OMIT			0.9	0.4

Overall performance on the perimeter and/or area items was significantly less than that on length or time or money.

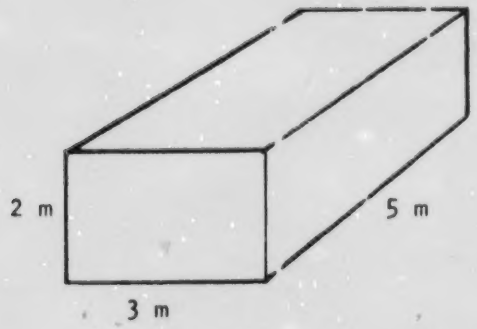
V.4 Volume and Capacity

Finding the volume of a rectangular solid gave rise to performances of __, 42, 75, 84. The related item is displayed in Table V-7 on the next page. Distractor A, which involved adding the three magnitudes (2 + 3 + 5) instead of multiplying them, is interesting in that 36% of the Division II students chose it. The percentage fell off to 11% for Division III and to 3% for Division IV.

Table V-7: Ex 351

What is the VOLUME of the rectangular solid?

A) 10 m^3
B) 25 m^3
C) 30 m^3
D) 40 m^3
E) I don't know



DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A		36.1	11.4	3.0
B		9.2	3.5	4.3
* C		42.5	74.6	83.7
D		6.8	6.1	2.6
E		5.0	3.7	5.0
OMIT		0.4	0.7	1.4

Recognizing that doubling the linear dimensions of a cube would have an 8-fold effect on volume, proved difficult with only 47% of Division IV students giving the correct answer. On the other hand, recognizing that people buy milk by the litre was noticed by 85% of Division II students. Table V-8 below displays the related exercise.

Table V-8: Ex 251

In Canada where the metric system is used, people buy milk by the

A) metre
B) litre
C) quart
D) gram
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	5.8	1.2		
* B	55.1	84.9		
C	25.8	12.3		
D	5.5	0.8		
E	5.0	0.6		
OMIT	2.8	0.2		

V.5 Mass and Temperature

The three exercises involving mass or temperature were aimed at tapping the students' familiarity with the appropriate units. Exercise 188 is typical and is shown in Table V-9 below. Typical also are the levels of performance ranging from 32% at Division I and rising to 89% at Division IV. It is somewhat surprising that after two years of hearing, seeing, and feeling the temperature of a sunny summer day, 11% of grade 12 students still don't associate 25° C with such conditions. Interesting among the alternative responses is 85° C which distracted 31% of the Division I students, decreasing to 3% at Division IV.

Table V-9: Ex 188

The temperature on a sunny summer day would most likely be

A) 5° C
B) 25° C
C) 55° C
D) 85° C
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	6.4	4.5	2.8	2.0
* B	31.6	45.8	78.3	89.4
C	22.3	24.9	8.3	4.6
D	31.0	22.3	8.5	3.2
E	6.9	2.4	2.1	0.6
OMIT	1.8	0.2	0.0	0.4

The results for the questions involving mass were quite similar to those for temperature.

V.6 SI Relations

Reported here are exercises dealing with relationships among and between SI units and sub-units. Of the three exercises given on this topic, the highest levels of performance were obtained on the exercise shown in Table V-10 below dealing with the relationship between centimetres and metres.

Table V-10: Ex 252

The number of centimetres in one metre is

A) $\frac{1}{100}$
B) 10
C) 100
D) 1000
E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	19.2	8.2	5.3	5.3
B	14.8	11.0	5.3	6.0
* C	40.3	60.1	72.9	75.0
D	8.1	17.6	13.3	11.6
E	15.9	2.0	1.8	2.0
OMIT	1.6	1.2	1.3	0.2

The other two exercises dealt with the relationship between cm^2 and mm^2 on one hand and cm and mm on the other. In both cases, performances were approximately 10% lower than on exercise 252 shown in Table V-10.

VI Statistics

The cluster analysis for Statistics follows.

	I	II	III	IV
Knowledge				
Comprehension	65 (3)	48 (3)	86 (2)	72 (2)
Application	35 (1)	(0)	66 (1)	14 (1)

Figure VI-1 Cluster Analysis for Statistics

This matrix shows that there were no strictly Knowledge items in the test. Most items were Comprehension with single Application items in three cells. The responses follow the general pattern of Application questions being more difficult than Comprehension items.

The thirteen statistics questions were divided more or less equally among the four Divisions with about half being uniquely targeted and the other half common. Although the content breakdown for statistics includes nine categories, only three were used in this assessment and ten of the thirteen items fell into the area of graphic representations.

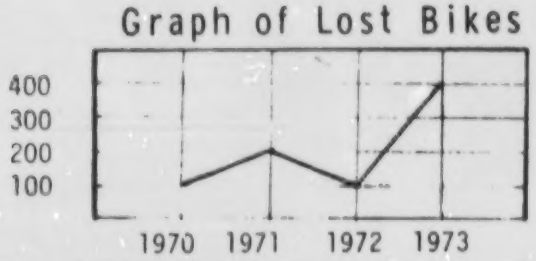
VI.1 Graphic Representation

The four items in Division I covered interpretations of picture graphs (89%), bar graphs (65%), and line graphs (41%), and a bar graph to be constructed given data and a graph format (35%). The line graph results show this graph to be understood by 93% of grade 3 students.

Table VI-1: Ex 191

How many bikes were lost in 1971?

A) 100
B) 200
C) 300
D) 400
E) I don't know



DISTRIBUTION OF RESPONSES

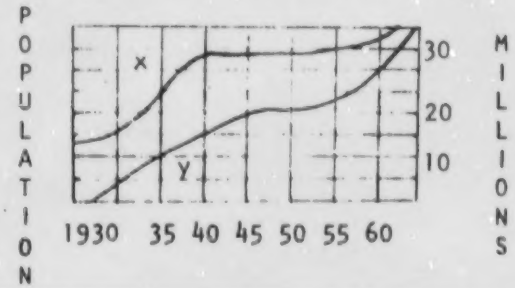
RESPONSE	DIVISION I	II	III	IV
A	18.1	5.2	3.6	
* B	40.9	88.6	92.9	
C	7.9	1.6	0.2	
D	18.4	3.8	2.3	
E	13.9	0.2	0.0	
OMIT	0.9	0.6	1.0	

The two Division II questions in this category covered recognizing ordered pairs plotted on a graph (12, 33, __, __) and the following item in interpreting a curved line graph with two distinct plots.

Table VI-2: Ex 254

The least difference in population between the two countries (X and Y) occurred in

A) 1930
B) 1940
C) 1950
D) 1960
E) I don't know



DISTRIBUTION OF RESPONSES

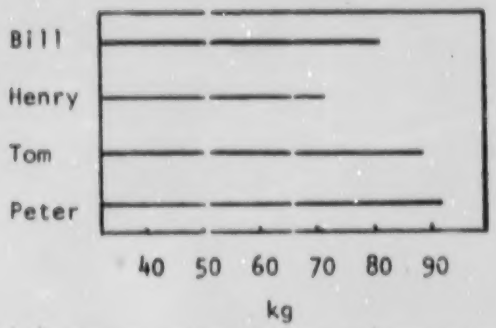
RESPONSE	DIVISION I	II	III	IV
A		22.7		
B		17.6		
C		5.1		
* D		47.5		
E		6.7		
OMIT		0.4		

At Division III, simple interpretation of a circle graph resulted in a performance of __, 52, 79, 90. A common item on bar graphs yielded results similar to one given at Division I.

Table VI-3: Ex 355

Which boy has a mass closest to 80 kg?

A) Bill
B) Henry
C) Tom
D) Peter
E) I don't know



DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
* A	56.0	83.6	92.3	94.6
B	10.3	6.1	6.2	2.5
C	22.4	3.5	0.8	1.5
D	6.7	5.7	0.4	1.2
E	3.0	1.0	0.4	0.2
OMIT	1.6	0.2	0.0	0.0

Only one Division IV item was given in this category. The question involved making a one-step generalization from a line graph. The results were __, __, 79, 92.

VI.2 Probability and Central Tendency

The results of the probability question are given below.

Table VI-4: Ex 454

At the start of a party game, eight red, six green, four blue, and two white slips of paper were thoroughly mixed in a bowl. The chances that the first slip drawn at random will be a WHITE one are given by which one of the expressions below?

A) $\frac{2}{8+6+4}$

B) $\frac{1}{8+6+4+1}$

C) $\frac{1}{8+6+4+2}$

D) $\frac{2}{8+6+4+2}$

E) I don't know

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				17.9
B				8.1
C				18.0
* D				52.8
E				3.2
OMIT				0.1

A Division II item on averages is given below.

Table VI-5: Ex 255

If 5 articles cost \$14, \$17, \$21, \$22, and \$26, the average cost per article is

A) \$18
B) \$19
C) \$20
D) \$22
E) I don't know

DISTRIBUTION OF RESPONSES

	DIVISION		
RESPONSE I	II	III	IV
A	10.6	4.0	2.7
B	7.0	6.2	1.8
* C	62.7	84.8	91.5
D	14.2	2.5	3.3
E	5.0	2.6	0.7
OMIT	0.6	0.0	0.1

And finally an Application item which called for a distinction among mean, median and mode scored very low (14%) at Division IV.

In summary, Division I scored high on the picture graph. Division II scored high on straight line graphs and bar graphs but scores were lower on curved line graphs requiring one step generalizations. Division III and IV did well on the graph questions, but Division IV scored only 50% on a simple probability question and scored low on applying elementary statistics concepts.

VII Consumer Mathematics

The assessment at the Division IV level was titled General Proficiency in Mathematics. General Proficiency was to include skills in the application of mathematics to consumer related problems.

To assess this aspect of mathematics, 18 consumer related items were included in the Division IV battery. These 18 items were not used in any other division in the assessment. All 18 items in consumer mathematics were at the Applications thought level. They required a solution to a verbally stated problem. The 18 items had a collective percentage of 51.

The sub-topics used in choosing items are as follows:

1. Comparative Shopping
2. Profit and Loss
3. Banking
4. Interest
5. Consumer Credit
6. Payroll
7. Taxation
8. Insurance
9. Stocks, Bonds and Investments
10. Real Estate
11. Discount

VII.1 Comparative Shopping

Two items were used in this category. The first of these is shown below in Table VII-1.

Table VII-1: Ex 456

A housewife will pay the lowest price per gram for rice if she buys it at the store which offers

A) 320g for 40 cents
B) 400g for 45 cents
C) 800g for 85 cents
D) 900g for 99 cents
E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				38.5
B				7.0
* C				34.7
D				16.4
E				2.8
OMIT				0.6

Although all comparative shopping problems are solved the same way, the percentages for the two items differed significantly. The item given above may have been particularly troublesome because it involved fractional cents per gram in solving. In contrast, the second item involved the purchase of tennis balls, each of which cost some whole number of cents. The percentage for this item was 69. Finally, it is interesting to note that students did not seem to think that the largest quantity was automatically the best deal.

VII.2 Profit and Loss

One item in this category required the computation of a selling price given the cost of an article and the desired margin of profit. The percentage for this item was 2. In most of the cases, the student multiplied the cost of the article by the margin. Problems involving margin of profit are not part of the curriculum so they are not learned apparently.

VII.3 Banking

One item in this category required the computation of the cost in Canadian dollars of purchasing traveller's cheques in United States currency. This item had a percentage of 18. This would seem to reflect a lack of familiarity with traveller's cheques.

VII.4 Interest

Two items were given in this category. One of these items is given in Table VII-2 below.

Table VII-2: Ex 461

Find the principal, if the simple interest received after two years at an annual rate of 6% is $60.

A) $2000
B) $5000
C) $ 500
D) $ 720
E) I don't know

DISTRIBUTION OF RESPONSES

		DIVISION			
RESPONSE	I	II	III	IV
A				13.4
B				6.3
* C				39.1
D				21.4
E				19.6
OMIT				0.2

Apparently problems involving simple interest are not given much emphasis in the present curriculum. Even Math 30 students had a percentage of only 48. Math 25 students had a percentage of 36. The second item in this category concerned the calculation of interest paid on a credit purchase of a car. The percentage for this problem was 70. This is obviously a problem that is in more familiar territory for these students.

VII.5 Consumer Credit

Two items were included in this category. One of these items is shown below in Table VII-3.

Table VII-3: Ex 462

Find the closing balance obtained for this bank credit card monthly statement. On unpaid balances, the bank charges 1½% interest per month.

Prior Balance	Payment	Purchases	Interest	Closing Balance
$80	$20	$84.27	$0.90	

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
RIGHT				13.0
WRONG				85.5
OMIT				1.5

Knowledge of basic terms that appear on such monthly statements seems to be lacking. The second item in this category is given in Table VII-4 below.

Table VII-4: Ex 463

A family buys a TV set on a "no down payment" 15-month installment plan. The 15 installments are equal. The price is $400. When the set is paid for, the family has paid $460, plus a repair bill. They trade the TV set in on a new one at the end of 15 months, and receive a $100 trade-in allowance.

What other information must be given before one can figure out the average monthly cost of owning the set for 15 months?

A) Amount of the dealer's profit

B) Average monthly payment

C) Annual interest rate

D) Amount of the repair bill

E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				1.5
B				13.4
C				21.8
* D				59.4
E				3.0
OMIT				0.8

This item makes a large demand on reading skills. It should also be noted that the percentage for boys was 13 points higher than for girls.

VII.6 Payroll

Two items were included in this category. One of these is shown in Table VII-5 below.

Table VII-5: Ex 464

Find the total wages for the following

Hours Worked		Wages per Hour	
Regular	Overtime	Regular	Overtime
40	10	$5.10	Time-and-one-half

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
RIGHT				66.4
WRONG				32.7
OMIT				0.9

Performance on these two items was higher, with percentages of 66 and 63 for the two items. Computing wages is a more familiar problem for these students.

VII.7 Taxation

Three items were given in this category. One of these items is shown below in Table VII-6.

Table VII-6: Ex 466

For property taxation, a city set its mill rate at 29.87. The tax on property assessed at $14,900 would be closest to which amount?

A) $400

B) $420

C) $450

D) $470

E) I don't know.

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A				5.2
B				13.0
* C				34.7
D				24.6
E				22.2
OMIT				0.4

Computation of property taxes using mill rates is an important consumer skill. It is obviously not automatically learned. The second item required the computation of a sales tax. Its percentage was 74. The third item required the reading of a tax table to find the total tax (federal and provincial) on a given income. It had a percentage of 80.

VII.8 Insurance

The one item in this category required the computation of a life insurance premium using a fee table. It had a percentage of 34. A complicating factor in this problem was the life insurance amount of $6,500.00 while the table was given in multiples of $1,000.00.

VII.9 Stocks, Bonds, and Investments

The one item used here involved the computation of an agent's commission on an investment sale after a 150% increase in value. Its percentage was 64. This is a reassuring indication of some skills in using percentages. Finally, it should be noted that the percentage for girls was 20 points lower than for boys.

VII.10 Real Estate

The item in this category required the computation of a monthly mortgage payment. It is given in Table VII-7 below.

This percentage probably reflects the familiarity of these terms and procedures. It should also be noted that the percentage for Math 25 was 17, while for Math 30 was 63.

Table VII-7: Ex 471

You are a new homeowner and you have signed a home loan agreement. You agree to make monthly payments to pay off the loan and to provide a reserve for paying the insurance and taxes. Find the total monthly payment if the monthly loan payment is $185 with annual taxes of $480 and a three-year insurance premium of $180.

DISTRIBUTION OF RESPONSES

	DIVISION			
RESPONSE	I	II	III	IV
RIGHT				46.8
WRONG				52.9
OMIT				0.3

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VII.11 Discount

The two items in this category required the computation of prices of goods given the regular price and the percentage discount. The percentages were 85 and 72, reflecting an understanding of discount problems and a skill in using percent.

VIII Other Variables

Reported in this section are analyses of performance as related to demographic/personological variables: (1) sex, (2) school size, and (3) mathematics program. As well, the results of the student background questionnaire are reported.

VIII.1 Sex

Sex, as a variable related to differential mathematical performance, has been the topic of many research studies. Over the years, it has become an accepted belief that males achieve better in mathematics than females (Glennon and Callahan, 1968; Maccoby and Jacklin, 1974). As well, the superiority in mathematics. Recent research (Fennema and Sherman, 1977) however, casts some doubt on the apparent male superiority, claiming instead that other variables of a socio-cultural sort have a great deal to do with the apparent differences. The results of the present survey suggest that for Alberta, the claim of maie superiority in mathematics in 1978 is true. In the present study, the boys outperformed the girls at each Division. The differences were significant at the 0.01 level for Divisions II, III, and IV, but not at Division I. Table VIII-1 gives a somewhat graphic picture of the sex differences over the four Divisions. At grade 12, the differences are rather striking with boys outperforming girls on all six content areas. Research has also indicated that boys do better than girls particularly at higher levels of thought. Table VIII-2 corroborates this conclusion indicating as it does that boys do better than girls at all levels of thought and in particularly do increasingly better with increasing complexity of thought. Overall then, the picture of sex-related differences as they impact upon mathematics achievement in Alberta is very consistent with the larger American scene.

Table VIII-1

Sex Differences Over all Exercises
Taken at Each Division

Topic	Division			
	I	II	III	IV
Number			b	bbbb
Algebra	b*	b		bbbb
Geometry	b		bbb	bbbb
Measurement	b	bbbbb	bbbb	bbbbbb
Statistics	bb	b	bb	b
Consumer	n/a	n/a	n/a	bbbbbb

* Each "g" represents a 1% difference in favor of girls
Each "b" represents a 1% difference in favor of boys.

Table VIII-2

Sex Differences on Targeted Exercises
for Each Division at each Thought Level

Content/Division		Thought Level		
		Knowledge	Comprehension	Application
Number	I	g	b	gg
	II	gg	bbb	bbbb
	III	ggggg	bbbb	bbbbbbb
	IV	bbb	bbbb	bbbb
Algebra	I			
	II			
	III		bbbb	
	IV	bbbbb	bbbb	bbbbb
Geometry	I	gg		bbbbbbb
	II	g		bbbbbb
	III	b		bbbbbbbb
	IV	bbbbbbbbbbb		bbbbbbb
Measurement	I			bbbb
	II	bbbbbbbb	bbb	bbbbbbb
	III	bbbb	gg	bbbbb
	IV	bbbbbbbbbbb	bbbbbbb	bbbbbb
Statistics	I		bbb	
	II		gg	
	III		b	bbbbbbbbbb
	IV		bb	
Consumer	I			bbbbbb

Each "g" represents a 1% difference in favour of girls.
Each "b" represents a 1% difference in favour of boys.

VIII.2 School Size

In order to utilize the variable of school size, the sample of schools at a given grade level was classified as small (s), medium (m), or large (l). The division points for the categories differed with each grade level. Table VIII-3 gives the summary figures on the classification of schools at each grade level.

Table VIII-3

Distribution of Respondents According to School Size

Grade	School Size		
	Small	Medium	Large
3	(1-30) 906	(31-70) 1483	(71-?) 984
6	(1-30) 900	(31-70) 1083	(71-?) 1085
9	(1-40) 576	(41-101) 835	(102-?) 1783
12	(1-65) 824	(66-195) 900	(196-?) 1720

As with the variable of sex, the variable of school size had a decided effect on the survey results. In general, the small and medium schools outperformed the large schools. In order to get at the size variable average performance was calculated for each school size for each of the content areas for each Division. Thus, an average performance figure was obtained for small schools on number at Division I, for small schools on number at Division II, and so on up through large schools on Consumer items at Division IV. These averages were compared for each content area at each Division and the best and worst performance could be identified. For Division I, the "best performer" was the small school and the large school was the worst. Over the 120 items as a whole, the difference between these two school types is significant at the 0.01 level. For Division II, the medium school is the best, the large school the worst, and again the difference is significant at the 0.01 level. For Division III, the medium school is still the best performer but this time the small school is the worst. This difference is as well significant at the 0.01 level. For Division IV, the variable of school size was not significant. In order to get a better picture of these differences, Tables VIII-4 and VIII-5 were drawn up. These tables focus on the best and the worst performers respectively.

Table VIII-4 shows the best performers for each content area at each Division. Certain cells in the table were left empty if no one school size did better than the nearest rival size by 1% or more. The striking thing about Table VIII-4 is the complete absence of the large school. At Division I, the small school predominates; at Division II and III the medium school predominates; at Division IV the variable of school size seems less relevant.

Table VIII-4

School Size Differences as Related to the "Best Performance" over each Content Area at Each Division

Content	Division			
	I	II	III	IV
Number	s	m	mm	
Algebra	ss		mmmm	
Geometry	ss		mmm	s
Measurement			mm	
Statistics	ss	mm	m	
Consumer	--	--	--	

s (small)

m (medium)

l (large)

Each symbol in the table represents a 1% difference over the performance of the nearest rival size school.

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As concerns the "Worst Performance", Table VIII-5 gives the results. Striking here is the frequent occurrence of the large school coupled with the complete absence of the medium school. It seems that the large elementary school (enrolment of more than 70) is not the best as far as mathematics achievement is concerned. As well, the small junior high school is the least favorable. At the high school level, size appears not to make much difference. It should be realized that the above conclusions are on the basis of averages. There were some schools that were contrary to the above trends.

Table VIII-5

School Size Differences as Related to the "Worst Performance" for each Content Area at each Division

Content	Division			
	I	II	III	IV
Number		III	ss	
Algebra		III		
Geometry	II	II	l	
Measurement	IIII		ss	
Statistics	III	l	s	
Consumer	---	---	---	l

s (small)

m (medium)

l (large)

Each symbol represents a 1% difference under the performance of the next worse rival size school.

VIII.3 Grade 12 Mathematics Program Variable

At grade 12, the purpose of the assessment was to measure general proficiency in mathematics. This implied that the assessment not be tied to a particular mathematics curriculum at grade 12. This conception of the assessment was a response to three specific aspects of the mathematics program in the high schools. In the first place, by grade 12, the mathematical backgrounds of the students differ greatly, secondly, examinations for the assessment of student progress in the matriculation program are already in place, and, thirdly, this is the final opportunity to assess mathematical performances before many of the students enter the world of work and adulthood.

Although the assessment was to measure general proficiency, it was obvious that performances would still vary as a result of differences in mathematical programs. Consequently a decision was made to use student mathematical program as a variable in the analysis of the data. The mathematical programs were defined in terms of six alternatives that could be defined as last courses in mathematics at the high school level. These alternatives were: one mathematics course, Math 20, Math 23, Math 25, Math 30 and Math 33.

The results by mathematics course and content are shown in Table VIII-6 below.

Table VIII-6
Mathematics Program by Content Strand Summary

	One Math	Math 20	Math 23	Math 25	Math 30	Math 33	Significant Differences
Number	54*	76	62	53	87	74	4.5**
Algebra	43	61	48	44	77	58	2.7
Geometry	50	65	54	48	74	61	4.5
Measurement	50	65	56	50	77	64	5.5
Statistics	62	72	68	64	77	72	7.8
Consumer Math	38	48	38	41	60	43	4.5

* The number 54 indicates the mean percentage of correct responses to the exercises in the Number Content strand by the grade 12 students who have had one mathematics course in high school.

**The number 4.5 for the Number strand is the minimum columnar difference necessary in order to conclude, with 95 percent confidence, that there is a significant difference between two column entries. For instance, for the Math 20 and Math 23 students, the difference on Number, 76-62=14, is significantly different from zero because it exceeds 4.5.

It seems reasonable to assume that the performances for Math 30 students should be higher than for the rest of the grade 12 students. The Math 30 students are generally more capable mathematically, have had a mathematics course more recently, and, have had greater depth in their mathematical program. Beyond the Math 30 population, it would be expected that the Math 20 and Math 33 populations would be next best in performance. Further, the students from the remaining three courses would be assumed to do least well. For the most part, the results tend to corroborate these assumptions and expectations. There is very little that is surprising at that level of analysis. However, it is possible to take the analysis into a finer level of detail in order to discover more specific parts of the mathematics curriculum wherein students from the various courses seem to diverge and where they seem to converge in their performances.

This level of analysis is the focus for the remainder of this chapter.

Number

Mean percentages of correct responses differed only slightly across the mathematics program on Number exercises targeted at Divisions I and II. Percentages, across program, were more divergent as the exercises progressed to Divisions III and IV. In essence, performances diverge across mathematics programs as the exercises progress from being targeted at Division I to Division IV.

The greatest divergence occurs in exercises relating to integers, to ratio and percent, and to the real number system.

Algebra

In a pattern similar to that for number, divergence across mathematics program was greater as the exercises progressed from being targeted at Division I to Division IV.

The greatest divergence in performance occurred in exercises relating to coordinate geometry, to polynomial expressions, to polynomial equations and inequalities, to polynomial functions and to rational equations and inequalities.

Geometry

In contrast to the number and the algebra strands, the divergences in performance across mathematics program were approximately the same regardless of where the exercises were targeted. It might be surmised that the study of geometry is less specialized across mathematics program than is the number or algebra content strand.

The greatest difference in performance levels occurred on items related to similarity, to line relationships, e.g., perpendicularity and parallelism, and to polygons.

Measurement

For measurement, the divergences in performance across mathematics program were about the same for all levels of exercises. Again, measurement is apparently not specialized at the upper grades.

The measurement topics of SI metric, volume and capacity, and perimeter produced the greatest amount of divergence in performance levels across mathematics programs.

These conclusions should be accepted cautiously because of a smaller number of exercises under the measurement content strand.

Statistics

Of all of the content strands included in this assessment, statistics produced the smallest differences in performance across mathematics programs. In general, the students from Math 25, and Math 23, etc. seemed to more closely approach the level of performance of the Math 30 student. Again, it must be noted that this conclusion is based on a relatively small number of exercises.

Finally, it is worth noting that a large difference in performance did occur on one exercise. It required the computation of a probability.

Consumer Mathematics

As noted previously, the eighteen exercises in this section were all targeted at Division IV and were given only there. Also, they were all classed as Application exercises.

The largest differences in performances were on exercises related to computation of total mortgage payments, to reading insurance fee tables, and to computation of gross wages.

VIII.4 Student Background Information

Although the responses to the six background questions are self-explanatory, some generalizations are offered.

Calculators (see Tables VIII-7 and VIII-8). The percentage of students who indicate some familiarity with calculators are approximately 32, 40, 53, 75 at respective Divisions, while the percentage of those who use them at all in schools is 5, 5, 17, 66. We should also note that at Divisions III and IV, the percentage of students who use calculators in their mathematics classes every day is 5% and 35% respectively.

Attitude to Mathematics (see Table VIII-9). The percentages of students who have a positive attitude toward mathematics are 86, 86, 79, 75 across Divisions. Although a trend is clear, an even more definite pattern is evident in the "yes, very much" category: 47, 28, 20, 29.

When students compared mathematics to other subjects, a liking for mathematics shows as follows: 68, 62, 49, 44 and again the pattern is more obvious in the "favorite subject" category where percentages steadily decrease from 31, 11, 8, 6. This means that other subjects become relatively more appealing as the student progresses.

Value of Mathematics (see Table VIII-10). Here the responses to a positive value of mathematics is very high: 90, 96, 94, 88. Looking at the "very valuable" category a definite downward trend is clear: 70, 70, 50, 32. It is pleasing to see that only 2.2% of grade 3 students feel mathematics is "useless".

Class Time (see Table VIII-11). The assessment wanted to determine how broad is the range of time spent on mathematics at the different grades. At grades 3, 6, and 9, few schools spend less than 170 minutes per week. At grade 3, the spread on time spent ranges fairly equally from 170 minutes to more than 290. The same is true for grade 6 with an even greater number at more than 290 minutes (approximately 60 minutes per day). At Division III, half of the schools spend approximately 40 minutes per day while the other half has about 10 minutes more. The amount of time spent on mathematics decreases significantly from Division II to Division III.

In regard to student background factors of calculators, attitude, value and class time, no investigation was made to determine how these factors related to sex, size of school or program; nor was there any investigation into how these relate to achievement in mathematics.

Table VIII-7 Use of Calculators Outside School

Do you use a calculator outside of school?

A. Never
B. Approximately once a month
C. Approximately once a week
D. Almost every day

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	63.7	59.6	46.4	23.2
B	19.8	30.7	35.0	33.5
C	8.3	7.2	14.7	26.6
D	4.3	1.9	3.3	14.6
OMIT	3.9	0.6	0.5	2.2

Table VIII-8 Use of Calculators in Mathematics Classes

Do you use a calculator in your mathematics classes?

A. Never
B. Approximately once a month
C. Approximately once a week
D. Almost every day

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	92.3	94.8	82.5	32.1
B	1.6	3.5	7.4	11.6
C	2.6	0.7	4.7	18.7
D	0.7	0.5	4.9	35.3
OMIT	2.9	0.6	0.4	2.3

Table VIII-9 Attitude Toward Mathematics

Do you like mathematics?

A. No, I'm afraid of it
B. No, I hate it
C. Yes, a bit
D. Yes, very much

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	1.3	1.7	3.1	4.4
B	11.2	11.6	17.4	19.1
C	39.1	58.3	59.3	55.6
D	47.0	28.1	19.7	18.8
OMIT	1.5	0.4	0.5	2.1

Compared to other subjects you have studied in school, which of the following statements best describes your feelings about mathematics?

A. Math is my least favourite subject
B. Math is not included among my favourite subjects
C. Math is included among my favourite subjects
D. Math is my favourite subject

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	10.8	10.5	13.4	14.8
B	19.8	26.7	36.8	38.6
C	36.4	51.3	41.5	38.7
D	31.8	11.1	8.2	5.7
OMIT	1.1	0.4	0.6	2.3

Table VIII-10 Value of Mathematics

How valuable do you feel math is to people?

A. Useless
B. Not very valuable
C. Valuable
D. Very valuable

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	2.2	1.2	0.9	1.1
B	6.2	2.5	4.6	9.2
C	20.2	26.0	44.1	56.0
D	70.0	69.5	50.1	31.5
OMIT	1.4	0.5	0.3	2.2

Table VIII-11 Class Time Spent on Mathematics

How many minutes of class time on the average do you spend in your regular mathematics class per week?

A. 140 - 170 minutes
B. 170 - 210 minutes
C. 210 - 250 minutes
D. 250 - 290 minutes
E. 290 or more minutes

DISTRIBUTION OF RESPONSES

RESPONSE	DIVISION I	II	III	IV
A	4.7	2.3	3.9	
B	22.0	14.0	50.3	
C	32.7	34.3	41.7	
D	19.5	20.3	3.8	
E	18.5	28.7	0.0	
OMIT	2.6	0.4	0.3	

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